

# Representation of Heterogeneous Objects during Design, Processing and Freeform-Fabrication

Sanjay Rajagopalan<sup>1</sup>   Roger Goldman<sup>2</sup>   Ki-Hoon Shin<sup>3</sup>   Vinod Kumar<sup>4</sup>  
Mark Cutkosky<sup>2</sup>   Debasish Dutta<sup>5</sup>

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<sup>1</sup>CORRESPONDING AUTHOR: Center for Design Research, 560 Panama Street, Stanford University, Palo Alto, CA 94305-2232, Ph: 650-725-0160, Fax: 650-725-8475, email: sanjay@cdr.stanford.edu

<sup>2</sup>Center for Design Research, 560 Panama Street, Stanford University, Palo Alto, CA 94305-2232

<sup>3</sup>2225, G.G. Brown Lab, 2350 Hayward Street, Department of Mechanical Engineering & Applied Mechanics, University of Michigan, Ann Arbor, MI 48109-2125

<sup>4</sup>GE Corporate R&D, P.O. Box 8, KWD 241C, Schenectady, NY 12301

<sup>5</sup>2250, G.G. Brown Lab, 2350 Hayward Street, Department of Mechanical Engineering and Applied Mechanics, University of Michigan, Ann Arbor, MI 48109-2125

## Abstract

Solid Freeform Fabrication (SFF) processes build parts layer-by-layer, starting from an electronic (CAD) representation of the part. Most commercial processes work with a single homogeneous material. However, new processes, such as Shape Deposition Manufacturing (SDM), are able to handle heterogeneous materials with embedded components. To take full advantage of these processes, advances in both part representation and process planning are required.

The orienting, supporting and subsequent decomposition of object geometry into simple entities (e.g. layers) is a fundamental milestone in SFF. Recognizing this milestone, and constructing an exchange representation around it, enables new design support and process planning tools that streamline the prototyping of complex, functional parts. We propose a 2-tier scheme in which parts are mapped from a generic, or process-independent representation to an SFF-specific representation used for process planning. The generic representation encodes geometry and heterogeneous material attributes. The process-specific representation encodes the part orientation during fabrication, decomposed part and support structure geometry, build sequence constraints and Boolean precedence properties (for combine operations), in addition to the geometry, material information and allowable variability in material attributes. We illustrate the use of these representations and the design tools they enable by demonstrating the design, process-planning and fabrication of a flexible multi-material link suitable for use as a robot leg.

KEYWORDS: SFF, Heterogeneous, Representation

# 1 Introduction

Solid Freeform Fabrication (SFF), also referred to as Layered Manufacturing, is a new method of fabrication. In SFF, a part is built by depositing and (optionally) removing material, layer-by-layer, under computer control. The input to an SFF process-planner is a Computer Aided Design (CAD) model of the part. The part geometry is typically *decomposed* into an intermediate representation (e.g. layers) before fabrication can proceed. The key advantages of SFF processes are complete access to the interior of parts and the decomposition of complex three dimensional shapes into comparatively simple local geometries for process planning and execution.

Commercial Rapid Prototyping (RP) processes typically build parts from thin 2.5D layers, (resulting in stair-stepping of inclined surfaces). They also typically work with a single, homogeneous material for each part. However, new SFF processes are under development at several research laboratories that can fabricate parts with 3D layers<sup>1</sup>, heterogeneous material attributes (e.g. composition, microstructure [15]), and embedded components (e.g. pre-fabricated bearings, sensors or electronics [3]). It is widely acknowledged within the SFF community [11] [17] [20] that for these processes, the simple triangulated boundary geometry (STL format) that is currently used as the primary exchange medium between designers and manufacturers of SFF parts is inadequate.

SFF processes vary significantly in their operational details and capabilities (geometry, materials, precision etc.). However, they share the concept of a growth or build direction. This direction is typically determined by the physics of the material addition phase of the process. In many cases (e.g. stereolithography, selective laser sintering, shape deposition manufacturing) the deposition process is gravity assisted, which results in a vertical growth direction (usually the z-axis of the deposition machine). Before a part can be built with SFF processes, its *orientation* with respect to the process growth direction needs to be selected. The determination of build orientation is essentially the point at which a commitment to SFF is made. The selected orientation, along with the process capability, largely determines the decomposed geometry and the manufacturability of the part. In addition, for a given process, part quality parameters such as accuracy, surface finish and material properties are usually dependent upon the chosen orientation.

In this paper, we propose a two-tier representation scheme for 3D heterogeneous parts slated for Solid Freeform Fabrication. The first level of abstraction is generic (i.e., process and application-independent), and it improves upon current boundary geometry representations by enabling heterogeneous material attribute specification throughout the interior of a part. This representation

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<sup>1</sup>Stacking up 3D layers yields the true object, whereas stacking up 2.5D layers results in a stair-stepped approximation of the object

extends the  $r_m$ -set and  $r_m$ -object models proposed earlier in [12] [13]. The second level of abstraction is a process-specific (or more accurately, a process-class-specific) representation that includes the specification of a part orientation with respect to the process growth direction. The oriented geometry in this representation is decomposed in a manner that facilitates process planning and enables design support tools (see Section 2.1.1) to be applied. It is widely believed that the growth and widespread use of VLSI technology was aided by parallel developments in exchange representations and design support tools [18] [26]. We believe that the widespread adoption of SFF is predicated upon similar developments in the mechanical design arena, which is the primary motivation for this work.

To illustrate the concepts advanced in this paper, we have taken an example part and carried it through the entire design-fabrication cycle (see Section 4). The example is a geometrically simple mechanism to be used in a small robot with flexible joints. The design calls for heterogeneous material attributes within the bulk geometry. This design is modeled using the generic representation, and subsequently oriented, decomposed, planned and fabricated. An alternate design method, wherein the oriented model is directly constructed using pre-decomposed library elements (thereby significantly reducing the process-planning burden) is also demonstrated.

## 2 Background

### 2.1 Design and Layered Fabrication of Heterogeneous Parts

Heterogeneous materials (also called Functionally Graded Materials - FGMs ) have become a subject of research in the Material Science, Composites, Ceramic Engineering and Metallurgy communities [27]. Only recently, however, has design and layered fabrication of heterogeneous engineering parts been the focus of research [4] [13] - possibly because of the highly evolutionary nature of the field. Heterogeneous material prototypes have demonstrated significant improvement in the performance of high-temperature turbine blades, injection molding tools, smart structures, and mechanisms. They are especially effective in improving thermo-mechanical performance, increasing toughness and strength without compromising other properties (like hardness, wear resistance, corrosion resistance), reducing interfacial stresses between materials, and limiting crack propagation.

### 2.1.1 Design and Process Planning

To create parts with heterogeneous materials the design specifications must be converted into representations suitable for SFF process planning. There are two basic approaches that designers can use: *design decomposition* and *design-by-composition*. We briefly describe each of these approaches in turn (see Figure 1).

In the *design decomposition* approach, the CAD model is analyzed to determine a build orientation and decomposed into simpler geometries that correspond to steps in the processing cycle. Additional process planning tasks include the generation of support structures (in order to support overhanging features), and planning of the path required to deposit material along and within each contour [17]. At present, design decomposition and process planning are not fully automated for complex parts. New methods for process planning are being developed to operate on heterogeneous solids and generate the required information for the SFF machine [4] [5] [10].

In the *design-by-composition* approach the designer creates the design using a library of predefined components. This approach is analogous to VLSI circuit design, where microprocessors are composed from transistors, gate arrays, etc. In this case, the components are already represented in process-specific terms, with a build direction and decomposition defined. A set of rules [2] ensures that as new elements are merged into the design, the material attribute and precedence constraints are preserved. Thus, the overall process plan for the part evolves in parallel with the design.

The two approaches can also be combined. Given a suitable, common process-specific representation, a design can be decomposed and then further developed using elements from a pre-defined library.

Depending upon the capability of the process, the existing decomposition and composition algorithms [2] [23] have to be modified to handle graded regions in the part. For example, if the deposition process is able to mix the constituent materials within the specified tolerances, then no modifications are needed to algorithms for decomposing objects with homogeneous, multi-material objects. If, on the other hand, such “seamless” mixing is not possible (either due to physical constraints, or due to material compatibility reasons), the algorithms need to be modified. One possible option is to discretize the graded geometry into homogeneous regions such that the desired tolerance specs are met. We demonstrate this approach in Section 4, using the example of a plastic robot leg mechanism.

### 2.1.2 Heterogenous Part Fabrication

Several novel techniques for the manufacture of heterogeneous parts have been recently demonstrated, primarily in the research community [4] [5] [10]. Some examples of heterogeneous parts built at Stanford University are shown in Figure 2. The most popular materials for heterogeneous objects have been ceramics and metals [27], including Steel-Invar, Aluminum-Silicon Carbide, Titanium Carbide-Nickel/Molybdenum, and Nickel-Aluminum Oxide. Multi-material parts (e.g., Figure 3) have also been built, in which discrete volumes of a particular material or component are embedded within a surrounding structure of otherwise homogeneous polymer.

Fabrication techniques for heterogeneous objects can broadly be divided into two categories - *constructive processes* and *transport processes* [27]. Examples of constructive processes are powder densification, spraying, coating and lamination processes. Transport processes include heat or mass diffusion, centrifugal separation or settling. Solid Freeform Fabrication processes can, in general, consist of one or more of the constructive processes mentioned above.

In this work, we are primarily interested in constructive fabrication of heterogeneous parts using SFF processes like Shape Deposition Manufacturing. The unique issues that arise during the fabrication of these parts are material compatibility, controlled mixing and deposition, and resolution of functional grading. Fabricators of prototype parts using these processes have demonstrated “smooth” gradation of material for certain combinations of materials [4]. However, the smoothness is not a rigorously defined parameter, and has usually been taken to mean full alloying (as opposed to isolated islands of one material in a matrix of a second material).

## 3 A Representation for Heterogeneous Objects

Modeling of heterogeneous objects aims to incorporate spatially varying material attributes into the model, along with geometry. We introduce two new notions in this regard. The first is the capability to specify material attributes, such as *composition* or *microstructure*, depending upon the designer’s preference. The second is to incorporate a notion of *allowable variability* (or tolerance) into the material specification. Our representation builds upon earlier work [12] [13], which deals only with composition specification, with no notion of allowable variability. We summarize earlier work and propose further enhancements to the representation in the following sections.

### 3.1 Specification of Geometry

Current solid modeling methods describe an object by capturing its geometrical information. The mathematical space chosen for modeling the geometry is the Euclidean 3-space  $E^3$ , referred to as geometry space. Each point is represented as a geometric point  $\bar{x}$  in  $E^3$ . Thus, subsets of  $E^3$ , called  $r$ -sets [8] [24], are chosen to represent the geometry ( $\varepsilon$  in  $E^3$ ) of a physical object  $O$ .

An  $r$ -set is defined as a compact, regular, semi-analytic subset of  $E^3$  [8]. Geometric transformations can be applied to  $r$ -sets to transform them in  $E^3$ . Any two  $r$ -sets can be combined using a set of modeling operations called *regularized* Boolean operations, which obey closure properties on the interior of the pointset.

### 3.2 Specification of Material Composition

The material composition at each point ( $p \in \varepsilon \in E^3$ ) in a heterogeneous object is defined over a *material composition space*, which is represented as the vector manifold  $V$ . It is assumed that the material at each point in an object  $O$  can be described as a combination of a finite number ( $n$ ) of “primary” material types. Any particular composition can now be described as a material point  $\bar{\nu}$  within the manifold  $V$  in  $R^n$ , with each dimension of the space representing the volume fraction of one of the  $n$  primary materials, and all the volume fractions at any given geometrical point  $\bar{x}$  summing to unity. This can be described as follows:

$$V = \{\bar{\nu} \in R^n \mid \sum_{i=1}^n \nu_i = 1, \nu_i \geq 0\} \quad (1)$$

An object with heterogeneous material, then, can be described as an entity  $\tau$  in the product space  $T = E^3 \times R^n$ . The tuple  $\tau = (\varepsilon, \eta)$  represents the geometry ( $\varepsilon$ ) of the heterogeneous object as an  $r$ -set and the composition of the object as a material map  $\eta$  on  $\varepsilon$  (i.e.  $\eta = \eta(\bar{x})$  is the set of all material points corresponding to  $\varepsilon$ ). This tuple  $\tau$  is given the name  $r_m$ -set. A point in the  $r_m$ -set is represented as  $(\bar{x}, \bar{\nu})$ . Details of this method of composition specification are described in [13].

In the following section, we extend this concept to enable material attribute specification (e.g. composition, microstructure or any other material attributes of interest to the designer). We organize the representation into two categories, depending upon the specificity of the information encoded. Furthermore, we introduce the notion of specifying allowable variability in composition or property by utilizing a tolerance vector ( $\bar{\rho}$ ).

### 3.3 An Enhanced Representation

The following sections describe a new, enhanced representation for heterogeneous objects. The primary purpose of this representation is to provide a method for designers of heterogeneous objects to encode geometric and material attributes of interest, and communicate them effectively to downstream processes. It is important to emphasize that there is no mechanism within the representation that implicitly enforces *consistency* amongst the various attributes specified. A tighter coupling between the specified attributes (e.g. composition, microstructure and material bulk properties) requires the application of micro-mechanical modeling techniques and algorithms that are beyond the scope of this paper. Micro-mechanical modeling of heterogeneous materials is itself an active area of ongoing research [7] [9] [16] [28].

We propose a two-tier representation scheme - *generic* and *process-specific*. Each tier has the notion of an aggregate geometric entity that is built up from basic entities. Alternately, the object geometry in each tier is partitioned into a finite set of disjoint decompositions - but the “appropriate” decompositions for each tier of the representation are dissimilar. The generic representation (as the name suggests) is a general modeling abstraction. It is independent of any SFF-related concern. It is a useful representation for any application in which representation of spatially varying material attributes within a solid model is important (e.g. data visualization, finite-element analysis). The process-specific representation is used to model oriented, supported and decomposed objects slated for layered fabrication. The support generation and decomposition are required to be *legal* according to the fundamental constraints of SFF processes (see Section 3.3.2). It is a representation that enables quick manufacturability feedback, compositional design, and automated processing and fabrication of SFF parts. However, it is likely to be of little use beyond the SFF domain.

#### 3.3.1 Generic Representation

This representation extends conventional geometric modeling techniques to include information about heterogeneous material attributes. It is composed of two types of basic entities ( $r_m - set$  and  $r_p - set$ ) and two associated aggregate entities ( $r_m - object$  and  $r_p - object$ ). Each of these are described below:

**The Basic Entities:** An  $r_m - set$  (proposed in [13]) is a subset  $\tau = (\varepsilon, \eta)$  of the product space  $T = E^3 \times R^n$ , where  $\varepsilon$  is an  $r - set$  and  $\eta$  is a *composition map* defined on the composition manifold space  $V$ . Here, we propose an additional modeling entity, an  $r_p - set$ , which can be used to specify any attribute of the material (as an alternative to the composition) at each geometric point  $\bar{x}$  in  $\varepsilon$ .

Examples of material attributes are microstructure, or bulk properties such as elastic/shear moduli, coefficient of thermal expansion, thermal conductivity etc. An  $r_p$ -set is a subset  $\pi = (\varepsilon, \mu)$  of the product space  $P = E^3 \times R^k$ , where  $\varepsilon$  is an  $r$ -set and  $\mu$  is an *attribute map* defined on a attribute manifold  $M$ . The manifold  $M$  could be a vector or tensor space depending upon the particular attribute being modeled. For example, the attribute map  $\mu$  can represent  $k$  unique scalar bulk properties at each geometric point  $\bar{x}$  in  $\varepsilon$ . Clearly, material bulk properties are meso-scale attributes that cannot be defined meaningfully at each math point. So, the geometric point is assumed to be resolved to an appropriately small, finite region about each math point [7] [28]. To deal with this issue in practice, we introduce an explicit resolution parameter into the process-specific representation in Section 3.3.2.

In general, the composition or attribute maps are implicitly defined through functions  $f$  and  $g$ . This can be expressed as:

$$\begin{aligned} f &: (\varepsilon \in E^3) \rightarrow (\nu \subseteq V) \\ g &: (\varepsilon \in E^3) \rightarrow (\mu \subseteq M) \end{aligned} \quad (2)$$

This is a simplification of the the attribute modeling representation proposed in [14].

The only constraint that is imposed upon the material composition and attribute map of these entities is that of continuity. For example, the function  $f$  is typically required to be  $C^\infty$ . However, this condition can be relaxed depending upon the requirements of the specific application.

**The Aggregate Entities:** The  $r_m$ -object ( $\Gamma$ ) and  $r_p$ -object ( $\Pi$ ) are defined as finite aggregations of  $r_m$ -sets and  $r_p$ -sets respectively. Alternately, the aggregate entities can be thought to represent disjoint decompositions of the object geometry, with each element in the decomposition being an  $r_m$ -set or and  $r_p$ -set. The aggregations are denoted as follows:

$$\begin{aligned} r_m\text{-object} &: \Gamma = \{\tau_j\} = \{(\varepsilon_j, \eta_j)\} \\ r_p\text{-object} &: \Pi = \{\pi_j\} = \{(\varepsilon_j, \mu_j)\} \\ &\text{where } j \in J \end{aligned} \quad (3)$$

$J$  is a finite integer termed the *size* of the  $r_m$ -object or  $r_p$ -object, and it corresponds to the number of basic entities that makes up the aggregate entity. Typically, a larger size denotes greater complexity in the geometry and material specification of the object. In designing a heterogeneous object, a designer may choose to specify it either as an  $r_m$ -object, an  $r_p$ -object, or both (depending upon the downstream application). In general, the sizes of the  $r_m$ -object and  $r_p$ -object for the same physical object are different [14]. Once again, the issue of programmatically ensuring consistency

amongst different designer specifications for the the same object is beyond the scope of this paper.

There are three conditions that the aggregate objects have to satisfy. First, the  $r_m$  - *sets* and  $r_p$  - *sets* that compose the objects have to be geometrically pairwise interior disjoint. Second, the  $r_m$  - *sets* and  $r_p$  - *sets* in an object are minimal (i.e. two adjacent sets in an object cannot be combined into a single legal set), and third, the closure of the interior of all the sets in an object has to exactly equal the union of all the sets (i.e. there are no interior voids that are not explicitly modeled).

Transformation operations on  $r_m$  - *objects* and  $r_p$  - *objects* are a combination of regularized Booleans on the geometry of the constituent  $r_m$  - *sets* and  $r_p$  - *sets*, and combine operations on the associated material composition or attribute. The combine operations for material attributes on an object model are discussed in earlier work [13] [14].

### 3.3.2 Process-specific Representation

In this section, we present the entities that are used to describe oriented, supported and decomposed designs. These entities (called *SFF - compacts* and *SFF - objects*) can be thought of as extensions or “derived classes” of the heterogeneous modeling entities described earlier in Section 3.3.1. First, we explain the concept of a *compact*. A compact has been defined [22] [23] as the maximum 3D increment of geometry that can be fabricated within a single cycle of material addition and (optional) material removal. A compact can be considered a “logical” decomposition, as it is based on geometric and material constraints that are fundamental to the process (i.e. tool access, support of overhangs etc.), and not constraints that could change as the process evolves. Downstream processes may choose to further decompose compacts into simpler entities (e.g. 2.5D layers) depending upon their specific manufacturing constraints. The only geometric requirement that compacts satisfy is that they are convex monotones in the direction perpendicular to a prescribed growth axis (A simple solid is a convex monotone with respect to a line  $l$  if, for every line  $l_p$  orthogonal to  $l$ , the intersection of the solid with  $l_p$  is connected). The differences between the two representations (generic and process specific) are illustrated in Figure 4.

**The Basic Entity:** The basic entities are termed *SFF - compacts*. They are extended  $r_m$  - *sets* or  $r_p$  - *sets* that implement the convex monotone requirement described above. In addition to geometry and material information (which is inherited from the underlying  $r_m$  - *sets* or  $r_p$  - *sets*), compacts also have a unique growth direction, a build order field, a Boolean-precedence field, and a resolution vector. The use and utility of these additional fields is demonstrated later in Section 4.

An *SFF-compact* is modeled as a subset  $\mathcal{C}_t = (\tau, \bar{s}, o, b, \bar{\rho})$  of the product space  $C_t = T \times R^3 \times I \times I \times R^n$ , or alternately, a subset  $\mathcal{C}_p = (\pi, \bar{s}, o, b, \bar{\rho})$  of the product space  $C_p = P \times R^3 \times I \times I \times R^k$ , where  $\tau, T, \pi, P$  refer to  $r_m$ -sets and  $r_p$ -sets as described earlier.  $\bar{s}$  is a direction vector corresponding to the build orientation for which the *SFF-compact* has been produced. The positive integers  $o$  and  $b$  refer to the position of the *SFF-compact* in the build order, and the Boolean precedence for the *SFF-compact*, respectively.  $\bar{\rho}$  is the resolution vector that defines the allowable material variability (for the composition or attribute maps specified in  $\tau$  and  $\pi$ , respectively).

The only constraint on an *SFF-compact* (in addition to those inherited from  $r_m$ -set and  $r_p$ -set definitions) is that of convex monotonicity of the geometry of the *SFF-compact* orthogonal to the specified growth direction  $\bar{s}$ . In other words, an *SFF-compact* has to be a “legal” compact, as defined earlier in this section.

The composition and property specifications introduced in Section 3.3.1 (for  $r_m$ -sets and  $r_p$ -sets) are actually methods of specifying the *nominal* material information about the heterogeneous object. Clearly, even in the event that the constituent materials fully alloy with each other in all proportions, there could be limitations on the resolution with which real SFF techniques can process the materials. For example, it may be impossible for a deposition system to deposit a powder-mix with infinite precision in composition control. Here, we have introduced a notion of specification of allowable variability in material composition or attribute in *SFF-compacts* by using a tolerance vector  $\bar{\rho}$ . The field  $\rho_i$  represents the maximum (worst-case) variability in the composition of material  $i$  (in the case of an  $r_m$ -set) or attribute  $i$  (in the case of an  $r_p$ -set), that can be tolerated by the object without functional failure or unacceptable degradation in behavior. Alternately, it can also represent the minimum increment by which the volume-fraction of a particular constituent material or an attribute metric can be controlled in the actual fabrication process.

By introducing the notion of tolerance on the material specification, manufacturers can determine if the desired resolution of the material variation is within the capability of their process. Alternately, downstream service providers could optionally discretize the part geometry of smoothly graded regions (and deposit a different material into each discrete space) if continuous gradations of the given materials are not feasible. Figure 5 demonstrates this concept.

During the design of a heterogeneous object, the designer who specifies the tolerance parameter is essentially saying to the fabricator that it is adequate to control the material to where the actual part material is equiprobably situated anywhere in the *tolerance region* associated with the specification. This approach is similar to that of specification of worst-case location tolerances in geometric models. For example, if  $\rho_i$  denotes the allowable variability on material  $i$ , with a specified volume fraction

$\nu_i^{sp}$  at a given geometric point, then the actual (as fabricated) volume fraction  $\nu_i^{actual}$  at that point is limited by:

$$0 \leq \nu_i^{sp} - \rho_i \leq \nu_i^{actual} \leq \nu_i^{sp} + \rho_i \leq 1 \quad (4)$$

**The Aggregate Entity:** The aggregate entities are termed *SFF – Objects*. *SFF – compacts* do not have much significance in and of themselves, and are partial object descriptions. In order to represent a real object slated for layered manufacturing, they need to be assembled into a collection. Thus, an *SFF – object* is defined as a finite aggregation of *SFF – compacts*. An *SFF – object* is, in effect, an extended  $r_m$  – *object* or  $r_p$  – *object*, and is denoted as follows:

$$\begin{aligned} SFF - object : \Sigma_t = \{C_{t,j}\} = \{(\tau_j, \bar{s}_j, o_j, b_j, \bar{\rho}_j)\} \text{ or } \Sigma_p = \{C_{p,j}\} = \{(\pi_j, \bar{s}_j, o_j, b_j, \bar{\rho}_j)\} \\ \text{where } j \in J \text{ (finite integer)} \end{aligned} \quad (5)$$

Some constraints and dependencies are enforced between the geometry, material, orientation, build-order and Boolean precedence fields of all *SFF – compacts* that constitute a single *SFF – object*. A few are inherited from the definition of  $r_m$  – *objects* and  $r_p$  – *objects*. Constraints on an *SFF – object* are:

1. The *SFF – compacts* that constitute an *SFF – object* partition the *SFF – object* into a finite number of pair-wise disjoint regions

$$C_i \cap^* C_k = \phi, i \neq k, \forall i, k \in J \quad (6)$$

2. The closure of the interior of all the *SFF – compacts* in an *SFF – object* is exactly equivalent to the union of all the compacts (i.e., there are no extraneous “voids” within an object - internal cavities are explicitly modeled with material type set to “air”, for example).

$$\|\{\mathcal{I}(C_1) \cup \mathcal{I}(C_2) \cdots \cup \mathcal{I}(C_J)\}\| \equiv C_1 \cup C_2 \cdots \cup C_J \quad (7)$$

Where  $\|\cdots\|$  indicates closure over the pointset, and  $\mathcal{I}(\cdot)$  denotes the interior of an  $r$  – *set*.

3. All *SFF – compacts* within an *SFF – object* are similarly oriented (i.e. have the same orientation with respect to the process growth direction). While this requirement applies to most (if not all) SFF processes in use today, researchers are experimenting with the idea changing the part orientation during fabrication. Thus, fabricators could relax this constraint if the specific process allows multiple orientations while building a part.

$$\bar{s}_i = \bar{s}_k, \forall i, k \in J \quad (8)$$

4. An *SFF – object* is fully supported (or encapsulated in support structure) and no two *SFF – compacts* with consecutive positions in the build order have the same Boolean precedence  $b$  (i.e. the compacts are minimal as long as geometric and material restrictions on a solid are not violated).

$$b_i \neq b_{i+1}, \forall i \in J, i \neq \max(J) \quad (9)$$

5. The build order of *SFF – compacts* in an *SFF – object* is sequential, and decompositions that enforce cyclic ordering are invalid (i.e. they need to be further decomposed until cyclicity is destroyed).

$$\{(o_i > o_j) \mid (o_j > o_k)\} \Rightarrow (o_i > o_k), \forall i, j, k \in J \quad (10)$$

Adjacent *SFF – compacts* in an *SFF – object* can be identified by performing non-regularized intersections on pairs of compacts. The closure condition (2) ensures that all set-operatic intersections result in purely lower-dimensional entities. Any two compacts for which the intersection is not null are deemed as adjacent. The build order fields of the *SFF – compacts* in an *SFF – object* implicitly encode precedence ordering constraints that are imposed by the fabrication process. Compact Adjacency and Precedence Graphs [21] that can be used to make process planning and scheduling decisions by implementing graph traversal algorithms can thus be easily derived as-needed from these *SFF – objects*.

## 4 Example: Design, Processing and Fabrication of a Robot Leg

The previous sections propose representations that can be used to model a heterogeneous object, either in a generic form or in a process-specific form. Since the process-specific form has a partial manufacturing plan associated with the geometry, it becomes feasible to provide client-side manufacturability feedback and design tools that assist novice designers to quickly prototype complex parts. Furthermore, a design encoded in this form can be automatically fabricated with minimal planning on the fabrication side.

In this section, we consider the specific example of a flexible robot leg design, and illustrate the various design, processing and fabrication steps associated with using the approach outlined in this paper.

## 4.1 Motivation for the Design

The robotics community has become interested in biologically inspired or “biomimetic” approaches in robot design [1] [6]. Heterogeneous materials are rare in human-made products but ubiquitous in nature. By employing SFF fabrication it becomes possible to build robotic structures that, like their biological counterparts, achieve localized variations in hardness, stiffness and damping. It is also possible to embed discrete components such as sensors, actuators and micro-processors to create robust, multi-material “assemblies.” We have picked a small robot leg mechanism to illustrate how the representation, presented earlier, and the associated design tools can enable fabrication of biomimetic parts.

## 4.2 Design of the Robot Leg Mechanism

The prototype leg used as an illustration in this paper highlights, in particular, the benefit of allowing designers to specify heterogeneous material attributes. Figure 6 shows the concept line-sketch and corresponding CAD model for a 5-bar mechanism with flexible joints. This is an un-actuated prototype part, which is to be used for evaluating the geometry, bulk properties (e.g. hardness, elastic modulus), and mechanical properties dependent on part geometry (e.g. stiffness, fatigue strength) of the candidate design. It is proposed that the leg be built of polymer (e.g. polyurethane) and that flexure joints are preferred to pin-joints due to simplicity, ease of manufacturability, robustness and damping. While it is conceivable that this elastic mechanism could be built from a single material, the desired performance from the structural and the flexural regions is very different. The structural regions are required to be strong and stiff while the flexural regions are required to be compliant, damped and fatigue-resistant. Fabricating the entire leg from a single material would result in compliant structural regions, or brittle and failure-prone flexures.

The ideal solution requires varying material property (i.e. hardness) between different regions of the part. A smooth graded interface between the types of regions is also desired to reduce stress concentration areas between material interfaces. Thus, the designer chooses to represent the leg as a *SFF – object* with the underlying generic model as an *r<sub>p</sub> – object*. No specific composition map is specified, except for the requirement that the part be made from polymer materials.

The fabrication facility that receives the leg design could try and achieve the specified material attributes by mixing various polymers at different ratios during deposition. However, the mixing of polymers in arbitrary ratios is typically not feasible because it affects the chemistry of the curing

materials and produces unpolymerized regions within the object. Thus, some technique (see Section 4.4) needs to be applied in order to discretize the geometry of the graded regions based upon the specified tolerance parameter. However, this discretization is likely to create some new problems. Sudden material changes between the urethanes create regions of higher stress concentrations at the interfaces and require bonding between cured and uncured materials during processing. Furthermore, the number of process cycles required (for Shape Deposition Manufacturing) increases with the discretization of graded regions. Thus, some balance needs to be found between the complexity of the discretization pattern, and the process complexity. A process-specific representation also enables pre-encoding of such concerns into a library element (see Section 4.4).

### 4.3 Modeling of the robot leg

The robot leg concept can be encoded into a CAD representation in two ways (as described in Section 2.1.1) - using the generic representation or a process-specific representation. Each of these is addressed in turn:

#### 4.3.1 Creating the Generic Leg Model

A solid model of the whole part is created by extruding the cross-section of the part and then material information is added to indicate the various (homogeneous and graded) material regions. In this example, the solid model without the material map was created at Stanford University, using the AutoCAD platform. Next, it was exported to the ACIS *SAT* file format, and sent to the Heterogeneous Solid Modeler (HSM) software implemented at the University of Michigan. The HSM uses the  $r_m - object$  or  $r_p - object$  framework (described earlier) to model objects, and is implemented in C++ using the ACIS kernel. A simple GUI has also been implemented using Motif and OpenGL libraries.

In the HSM software, the user can optionally select standard modeling primitives (e.g. sphere, cone, block, etc.) and then input material functions for their interior. Then using standard Boolean operations these heterogeneous primitives can be combined to create heterogeneous objects (i.e. the  $r_m - object$  or  $r_p - object$ ). Alternately, it is possible to directly import the finished geometry (as in the case of the Stanford leg design), and add material information to it.

In the current implementation of HSM, only one material function can be assigned to each geometry. Since the robot leg contains 6 graded regions, the original geometry model is first partitioned

into 6 separate lumps. In order to assign the material map, the user can choose an appropriate coordinate type (cartesian, cylindrical, or spherical) and coordinate system (global, local, and user-defined) to define the origin and axes. For this example, a cylindrical coordinate is chosen for the segment in the middle junction and a cartesian coordinate for all the others (Figure 7). In addition, for the purpose of simple implementation, a linear material function was used for each of the six segments even though in the HSM, any mathematical expression can be used.

It is also possible to visualize the geometry and material function of the heterogeneous solid models within the HSM (see Figure 7). Note that the leg model created using this technique is independent of any process related constraints.

#### 4.4 Process Planning for the Robot Leg

When the leg design is encoded in the generic representation, any manufacturing process (SFF or traditional) that accepts the geometry and material information, in the chosen format, can be used to fabricate the part. The part could be processed in any orientation and in fact, the model can be used for any purpose including, for example, FEA, 3D-rendering etc.

For SFF, the  $r_p$  - *object* must be converted to an *SFF* - *object*. The model is first oriented, then supported and decomposed into a format suitable for layered fabrication. If the materials specified cannot be graded to an infinite resolution, some pre-processing algorithm needs to be used in order to set the resolution of the material deposition controller, or to discretize the geometry.

In the present case, the leg was built out of thermoset polymers IE70DC (Shore D Hardness 70) and IE90A (Shore A Hardness 90), manufactured by *Innovative Polymer Inc.*. These polymers could not be dissolved into each other, thus the graded regions needed to be discretized before they could be fabricated. The designer can control the resolution of the discretization by specifying the tolerance parameters on the material composition maps. Figure 8 shows two different discretization algorithms (one and two-dimensional) applied to the graded parts. In 1-D discretization (Figure 8(a)), the graded part is discretized in only one spatial dimension (defined on the local co-ordinate system). In 2-D discretization (Figure 8(b)), the graded region is discretized symmetrically along two spatial dimensions. Figure 9 shows the part sliced for fabrication in two different orientations. Note that one of these orientations would result in stair-stepping of the part surfaces (assuming 3-axis machining) and/or seams in the bearing surfaces, which could, in-turn, result in several design-manufacturing iterations to fix the problems.

#### 4.4.1 Composing a Process-Specific Leg Model

As an alternative to the above procedure, the model can be composed from a library of shapes that already have a partial manufacturing plan pre-encoded into them [2]. The merging algorithm automatically generates process plans for the aggregate entity based upon the Boolean precedence fields in each *SFF-compact* being merged. The composition of the graded regions is also pre-determined, based upon the material type and the process-capability for which the primitives were originally designed. Thus, to create the graded regions for the polymer leg, the designer adds the pre-discretized graded polymer primitives into the appropriate spots in the model. In using this technique, the designer is restricted by the available primitives and library elements that the design tool provides. The designer is also restricted in the transformations that can be performed on the primitives and elements. However, a part built using this technique is likely to be more manufacturable than one that is built without constraints. Thus, there is a tradeoff between flexibility and turn-around for the designer.

Figure 10 shows an exploded view of the process-specific model (*SFF-object*) of the polymer robot leg, including discretized regions that have been created to approximate the desired graded regions in the original design of Figure 6.

#### 4.5 Fabrication of the robot leg

The robot leg linkage is fabricated in two layers consisting of three individual polymer pours. While the deposition of some materials (such as powdered metals) has been automated at the Stanford RPL, thermoset plastics, such as the urethanes in the example leg, are deposited manually. The process cycles through three basic SDM steps: shaping of part substrate, deposition of part material and removal and shaping of deposited material. The shaping step of the cycle often becomes the initial step of the next process cycle in the fabrication sequence. Figure 11 illustrates the fabrication cycle for the robot leg. The final part built using this technique is shown in Figure 12.

### 5 Conclusions and Future Work

In this paper, a two-tier representation scheme for modeling heterogeneous material objects slated for Solid Freeform Fabrication has been proposed. The representation recognizes the orientation, support generation and decomposition steps as fundamental milestones in the fabrication of the

part. This results in the definition of *generic* and *process-specific* modeling entities that enable the designer to optionally choose between full-flexibility and rapid turn-around. The technique is demonstrated for the design, processing and freeform-fabrication of a flexible robot-leg part using the SDM process. While the techniques demonstrated work well with SDM, it is anticipated that the approach will extend to most other commercial and research SFF processes available today. The domain of applicability of SDM includes 3D geometry, non-planar layers, heterogeneous material composition and embedded components. Most SFF processes work on a subset of the SDM domain (i.e. 2.5D, planar layers, single material etc.), which the representations presented in this paper should capture.

Future work will include extending automatic orientation, support-generation, decomposition, tool-path planning and fabrication routines to accept object models encoded in these proposed formats. In addition, work is ongoing to create a library of primitives that can be used to build complex robotic devices with embedded components (joints, transducers and electronics) - with each primitive encoded as an *SFF - object* that has associated geometry, material, orientation, build-precedence, Boolean-precedence and material tolerance fields. It is hoped that these steps will make it possible for novice designers to rapidly create and fabricate complex, functional, electro-mechanical devices using Solid Freeform Fabrication.

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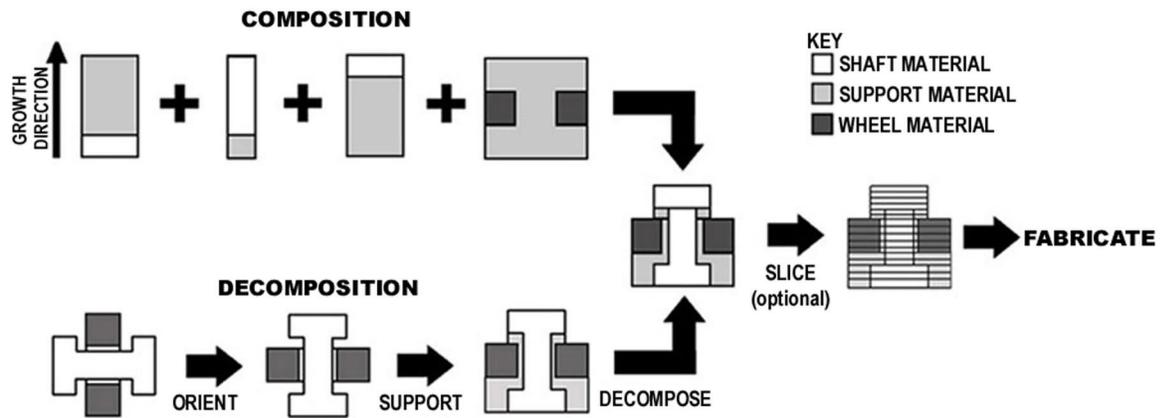


Figure 1: Two Design Styles - Decomposition and Composition

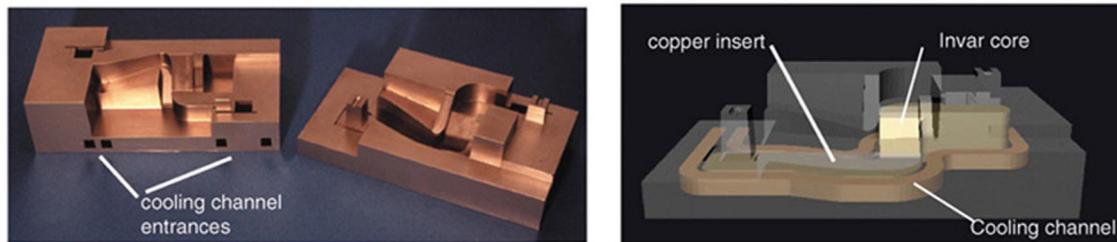
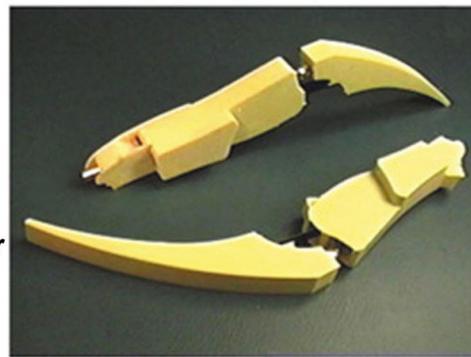
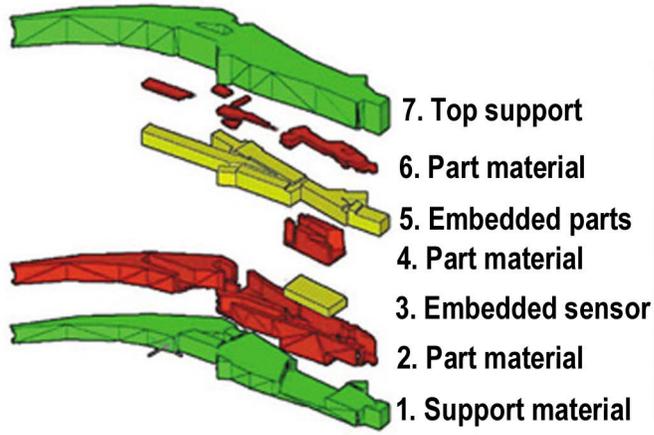
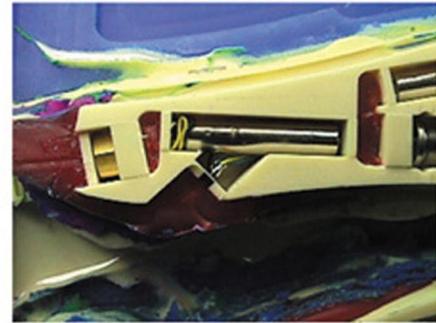
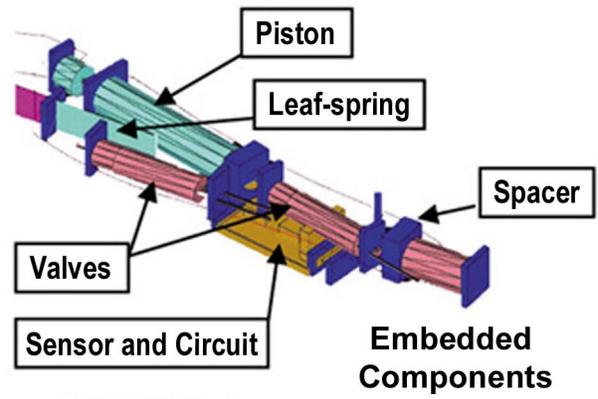


Figure 2: Photographs and CAD model of injection molding tool. Tool features an Invar core and stainless steel shell, with a graded material interface, embedded copper inserts and interior channels for cooling [4]



Sequence for Fabrication

Finished Parts

Figure 3: Pneumatic leg with embedded actuator, valves, sensor and circuitry [3]

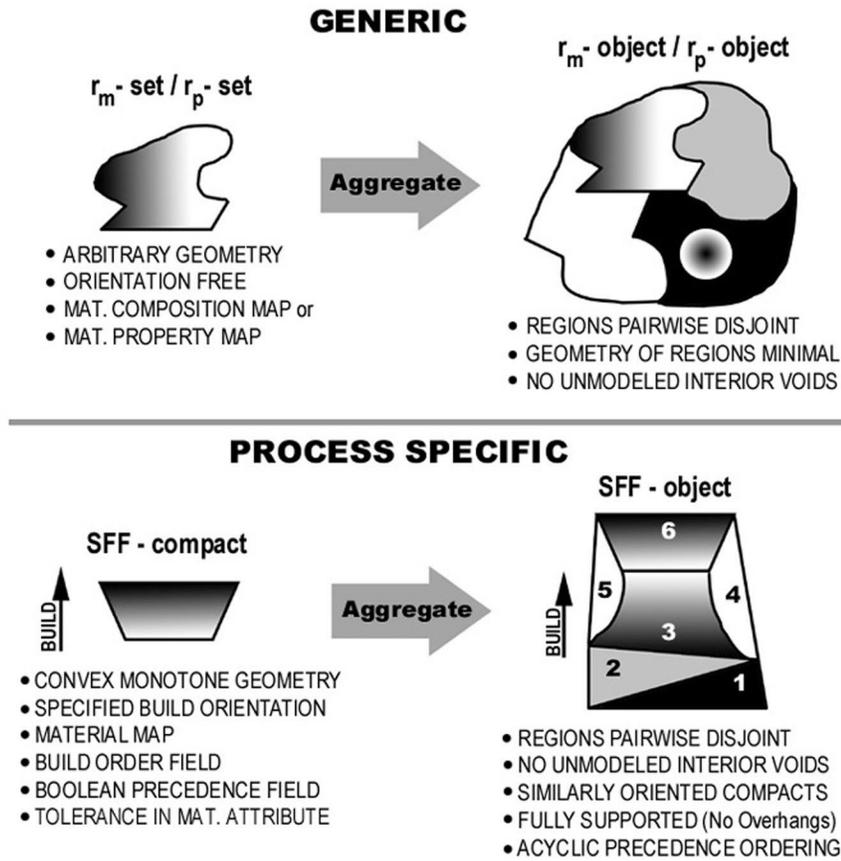


Figure 4: The Two-Tier Representation of Heterogeneous Objects

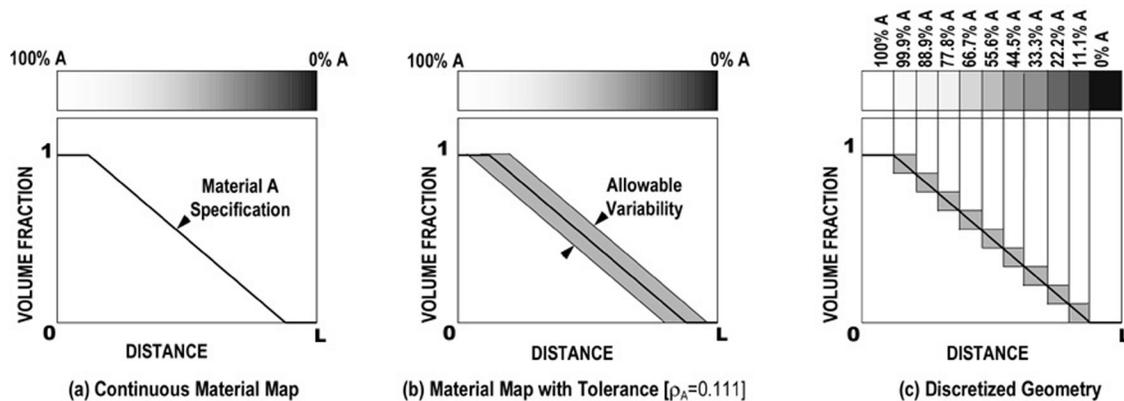


Figure 5: Discretization of a Continuous Material Map Using the Tolerance Parameter

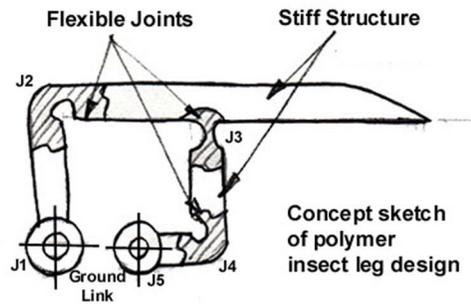


Figure 6: Concept sketch of robot leg design

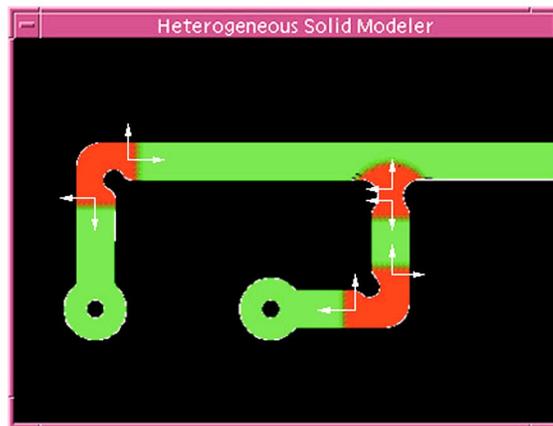


Figure 7: Material maps for the 6 graded sections on the insect-leg design

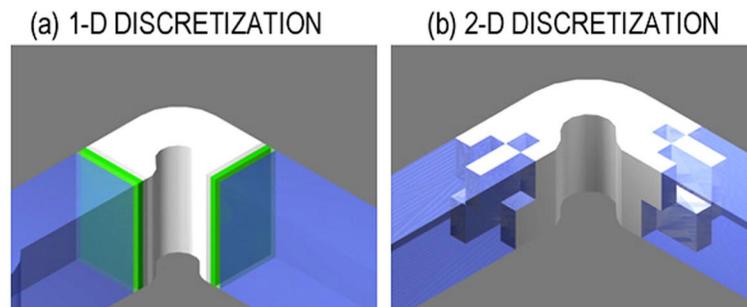


Figure 8: 1-Dimensional and 2-Dimensional Spatial Discretization of the Graded Leg Model

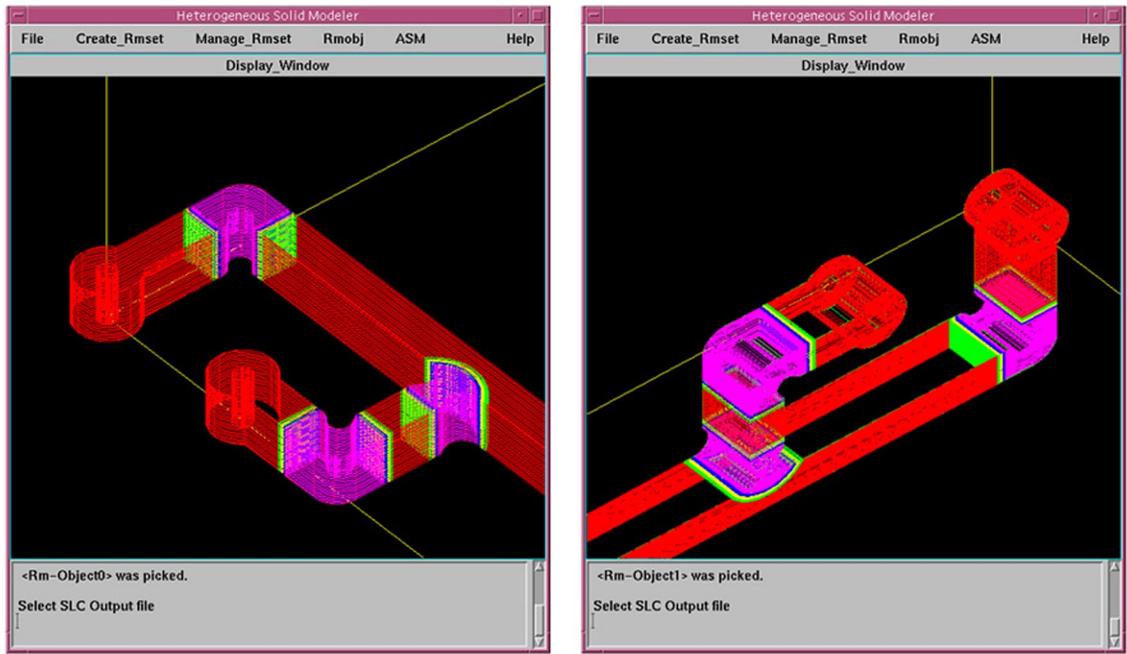


Figure 9: Leg Model Sliced in Two Orthogonal Orientations

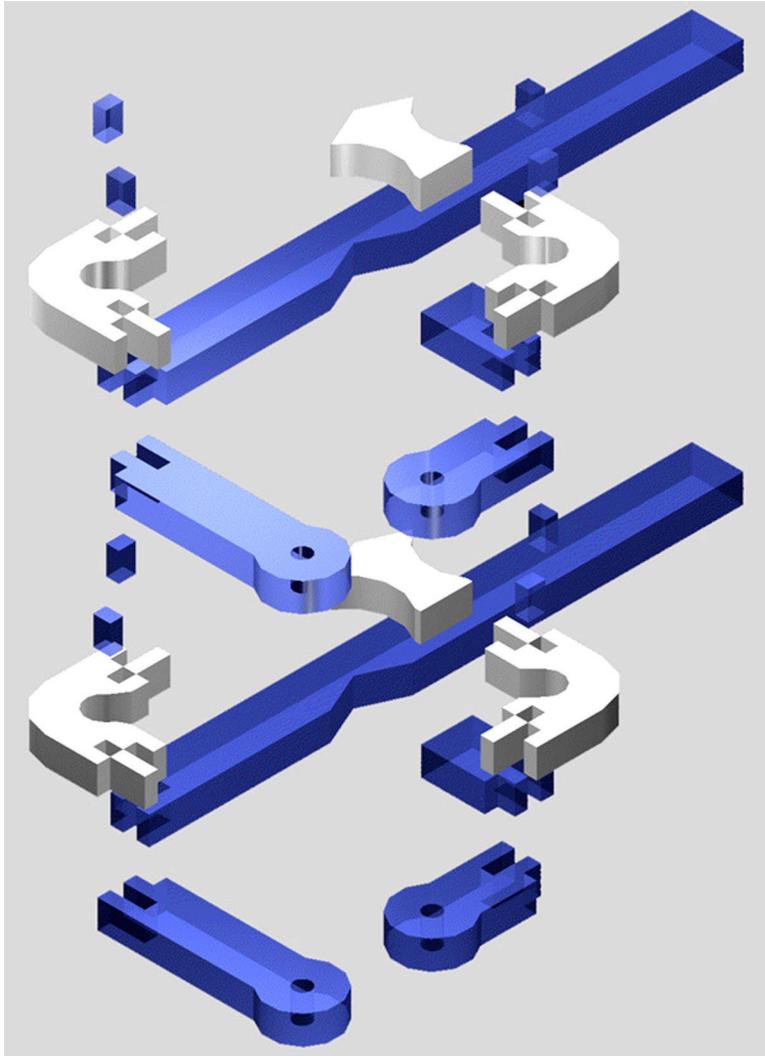


Figure 10: Process-specific design of the robot leg composed from primitives and pre-discretized graded regions

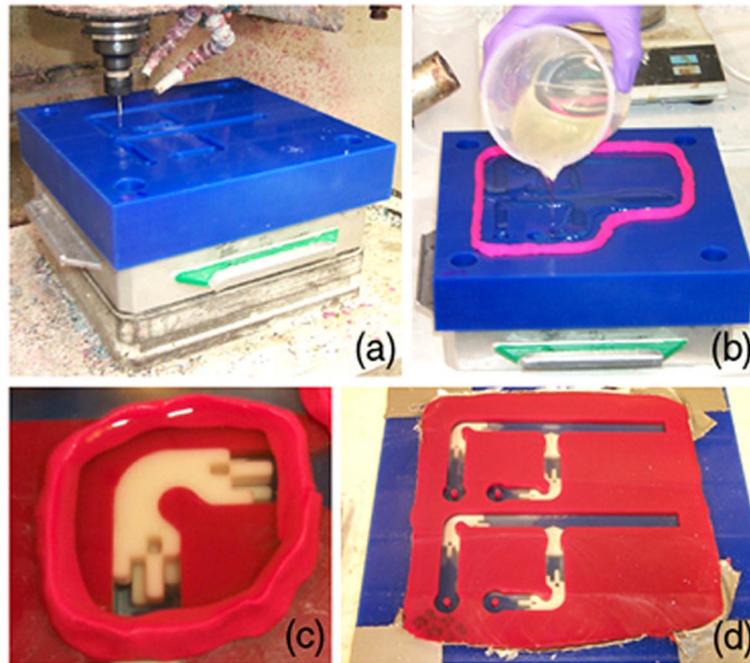


Figure 11: Fabrication of the robot leg at Stanford Rapid Prototyping Laboratory (RPL). Process Steps: (a) Machining blue wax (b) Pouring polymer (manual) (c) Closeup of deposited clear-plastic (d) Partially completed part on substrate

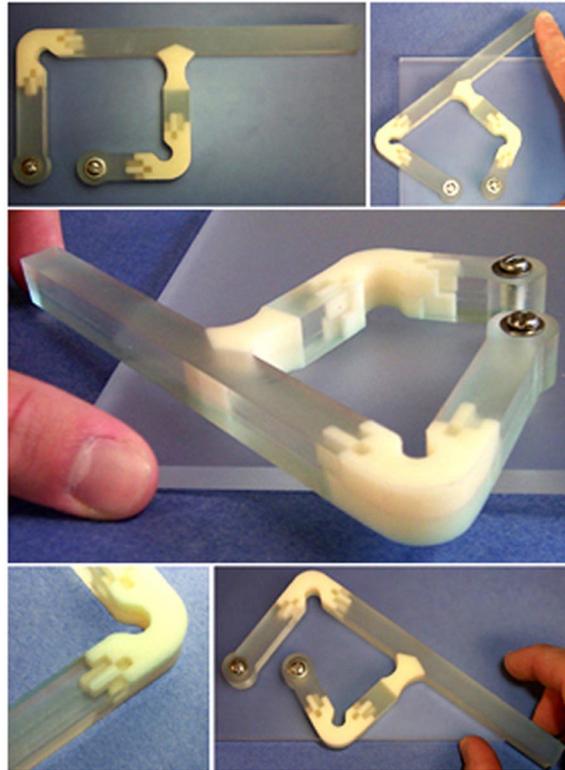


Figure 12: Final flexible robot leg