Proceedings of DETC'99
1999 ASME Design Engineering Technical Conferences
September 12-15, 1999, Las Vegas, Nevada, USA

DETC99/DFM-8958

OPTIMAL POSE SELECTION FOR IN-SITU FABRICATION OF PLANAR MECHANISMS

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ABSTRACT

Solid Freeform Fabrication (SFF) techniques allow the *in-situ* fabrication of fully-assembled devices with mating/fitting parts. Recently, this technique of fabrication has been found to be useful for building integrated mechanisms in robotics, and a wide array of other similar applications are anticipated. An interesting issue that arises during the fabrication of such mechanisms is the determination of an optimal *pose* in which the mechanism should be built. For example, should the mechanism be built in a folded or stretched-out position? In conventional manufacturing these issues do not arise, as each individual link is typically manufactured separately and then the pieces are brought together during assembly.

In this paper, we address the issue of finding a preferred (or optimal) pose for *in-situ* fabrication of planar mechanisms. There are many factors (e.g. achievable tolerances, non-interference, workspace size limitations, thermal considerations etc.) which can determine the suitability of a candidate build pose so that pre-specified task requirements are met. We limit our analysis to finding the optimal build configuration given achievable (in general, non-homogeneous, anisotropic) accuracy on joint position. For this treatment, we also make the simplifying assumption that the task requirements can be best satisfied by minimizing variability of link-lengths. Alternate task requirements, for example, maintaining end-point accuracy within a tolerance region, are being considered as part of ongoing work. We cast the problem of minimizing variability in link length as that of determining the relative position of two location tolerance regions for which the difference between their extremal distances is at a minimum (i.e., as they undergo constrained relative motion in the Euclidean plane). The method is similar to computational geometry techniques that have been developed in pattern matching and robot motion planning, with some important differences. We present some example mechanisms and their optimal pose under given workspace configurations.

NOMENCLATURE

$\delta(A, B)$	The Euclidean distance metric between two regions A and B
D_{max}	Maximum Euclidean distance between two planar regions
D_{min}	Minimum Euclidean distance between two planar regions
$\mathcal{R}_i$	Planar regions in Euclidean space
$\mathcal{S}_i$	Discrete planar point-sets
p_i, q_j	Candidate points in $\mathcal{R}$ or $\mathcal{S}$
θ	Orientation of a single link with respect to the fixed datum
θ_i	Link angles of a multi-link planar mechanism
s_θ, g_θ	General shape transformation operators
S_θ	Rigid body rotation matrix
Δ_{min}	Minimum difference between the maximum and minimum (extremal) link lengths at any single-link orientation
Δ^T_{min}	Minimum cumulative difference between extremal link lengths for all mechanism orientations

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INTRODUCTION

Mechanisms are important sub-components of many products and devices in common use today. Some prominent application areas are automobiles, robots, aircraft, satellites, micro-machines and other mechanical and electro-mechanical devices. Consequently, a thorough understanding of the science of designing and fabricating accurate and robust mechanisms cheaply is essential for mechanical designers. Recent advances in manufacturing technology, most importantly the advent of Solid Freeform Fabrication (SFF), or Layered Manufacturing, could revolutionize the manner in which these mechanisms are prototyped and fabricated (Cham *et al*, 1999)(Weiss *et al*, 1996). Unlike traditional manufacturing techniques, where the components that comprise a mechanism are individually fabricated and then assembled, layered manufacturing allows mechanisms to be simultaneously fabricated and assembled *in-situ*. This capability has some ramifications on the manner in which manufacturing variations affect the mechanism behavior.

This paper establishes a framework within which the relationship between achievable manufacturing tolerances and the kinematic behavior of mechanisms can be used to determine a preferred build configuration. The problem of studying the effects of worst-case and statistical tolerance stack-up on the positional accuracy of a mechanism with and without ideal joints has been addressed earlier (Tsai and Cutkosky, 1997)(Joskowicz *et al*, 1997). The reverse problem, where the manufacturing variations are determined given the measured positional accuracy of a mechanism, has also been addressed in the area of robotic calibration (Nahvi *et al*, 1994). In this paper we consider the problem of determining an optimal configuration (or *pose*) for the mechanism during fabrication, given certain assumptions about the geometry of the mechanism, the capability of the layered manufacturing process used to manufacture it, and the design objectives for the mechanism. At this time, we have restricted our analysis to planar mechanisms, as most SFF processes work best with 2.5D geometry, and hence are more suitable for building planar devices. Planar mechanisms also comprise a large proportion of the mechanisms being designed and used today in real products.

Our approach is to cast the problem as one of examining pairs of regions (or alternately, discrete point-sets) moving in a Euclidean plane. Achievable positional accuracy at build time is modeled as a pre-specified *tolerance region* of arbitrary geometry, in the neighborhood of the nominal position. It is important to emphasize that we are not addressing joint clearances (as in (Joskowicz *et al*, 1997)), but only the uncertainty in the position of ideal joints. The worst-case (minimum and maximum) link lengths are represented by the extremal distances between the tolerance regions of each joint. Various behavioral requirements can then be expressed as restrictions on the allowable variability of either links or link-chains. In this paper, we consider the simple case of maintaining the individual link lengths as close as possible to their designed length. The problem is then reduced to finding the configuration for which the difference in the extremal distances between the tolerance regions of adjacent joints is minimized, as the nominal joint positions (and hence their associated tolerance regions) undergo constrained motion in the Euclidean plane. Clearly, similar analysis would also apply to determining the best build pose for parts that have multiple critical dimensions among features (such as holes in a bolt circle).

Algorithms for determining the extremal distances between point-sets, and the associated complexity analysis, have been developed in the computational geometry community with regards to pattern matching (Huttenlocher *et al*, 1994)(Toussaint, 1984)(Bhattacharya *et al*, 1983) and robot motion planning (Latombe, 1991). The approach here is similar, with some important differences which we discuss in subsequent sections.

BACKGROUND

Solid Freeform Fabrication of Mechanisms

Solid Freeform Fabrication (SFF) is the name given to a family of recent fabrication techniques that first decompose the CAD models of parts into simple features (e.g. planar layers), and then build the part one decomposed feature at a time. The process can be thought of as a cycle of material deposition (or part embedding), and optional material shaping. Several variants of SFF processes exist in the commercial (e.g. Stereolithography, Selective Laser Sintering, Laminated Object Manufacturing) as well as research arenas (e.g. Shape Deposition Manufacturing, 3D Printing (Merz *et al*, 1994)(Sachs, 1992)). Most of the commercial processes are “feature-blind” (i.e. they operate on purely geometric entities like lines and curves, and are blind to the existence of design features like joints, shafts, sensors etc.). This characteristic, while very attractive from the process automation point-of-view, is largely responsible for restricting these processes to “look and feel” type of prototyping. In order to build functional engineering components that are highly integrated and precise, it is imperative that these processes allow for special handling of tolerance sensitive sub-components such as joints, sensors, actuators and electronics.

Figure 1. SOME MECHANISMS BUILT WITH SDM (Courtesy: Jorge Cham and Mike Binnard, CDR)

Shape Deposition Manufacturing (or SDM) is a process which allows for the optional embedding of functional components during the fabrication of a layered part (Cham *et al*, 1999)(Weiss *et al*, 1996). As an example, pre-fabricated joints with precision machined shafts and bearings could be inserted automatically into the partially-built work-piece during fabrication. In the most general scenario, the manufacturing process executes a "joint" subroutine given a position vector corresponding to the nominal location of the joint. This subroutine can either execute a library program that fabricates a high-precision joint from scratch, or it could signal a robot to insert a pre-fabricated joint at the specified position. The process of making a planar mechanism with multiple links is just a sequence of insertions of joints, with each insertion followed by the layered fabrication of the link bodies that encapsulate the embedded high-performance joints. This involves cyclic material deposition and material shaping steps. Once all the joints are embedded, the support structure is removed (typically by etching, melting or dissolving) to reveal a fully assembled functional mechanism with integrated joints. Figure 1 shows some mechanisms built using this technique. Some of the joints shown are pre-fabricated journal bearings with precision ground stainless steel shafts and teflon bushings.

Machine-tool and Process Accuracy

Most commercial manufacturers of Rapid Prototyping (RP) equipment have shied away from publishing specification sheets detailing the basic process accuracy. Unlike manufacturers of conventional CNC machines, who specify positioning and runout precision and repeatability numbers, RP manufacturers choose instead to provide information to potential customers about build envelope, minimum feature sizes, and material properties. This trend can be explained by the fact that the accuracy with which a layered object can be manufactured is influenced by many ill-understood phenomena (in addition to the machine-tool positioning accuracy), namely, shrinkage, warpage, stair-stepping etc. However, this has to change if these technologies are to be considered for use in building accurate functional components. At the very least, the manufacturers will need to provide dimensional (i.e. position and size) accuracy numbers for the process as a whole (which may be different from the positional accuracy of the machine tool used). Hybrid processes like Shape Deposition have a fundamental advantage that their dimensional accuracy is closely related to the accuracy of the material removal process used to shape the external features. In this paper, we only consider the locational accuracy with which pre-fabricated joints can be inserted into a workpiece in-process. Clearly, this will depend both on the positional accuracy of the machine-tool (robot or x-y table), and other factors such as part shrinkage, warpage etc. We assume that, in the general case, this accuracy is expressed as a tolerance region of arbitrary geometry around the nominal position of the joint within which the actual joint is equi-probably located.

Dimensions and Tolerances

It is important to distinguish the approach adopted in this paper from those of representing geometric tolerances on traditional mechanical designs. Recent work on Geometric Dimensioning and Tolerancing (GD&T) has focused on developing a rigorous mathematical foundation for the unambiguous representation of position, form and runout tolerances on mechanical designs (Requicha, 1983)(Srinivasan and Wood, 1995)(Jayaraman and Srinivasan, 1989)(Requicha, 1993). The issue of tolerance representation is also closely related to that of expressing manufacturing accuracy and process capability. However, we have chosen to use a very general model for expressing the positional accuracy of the manufacturing process for a number of reasons.

Traditional notions of specifying tolerances on manufacturing features (likes holes, shafts and pockets) are largely irrelevant for SFF. As mentioned earlier in Section , the greatest advantage of SFF processes still remains their rel-

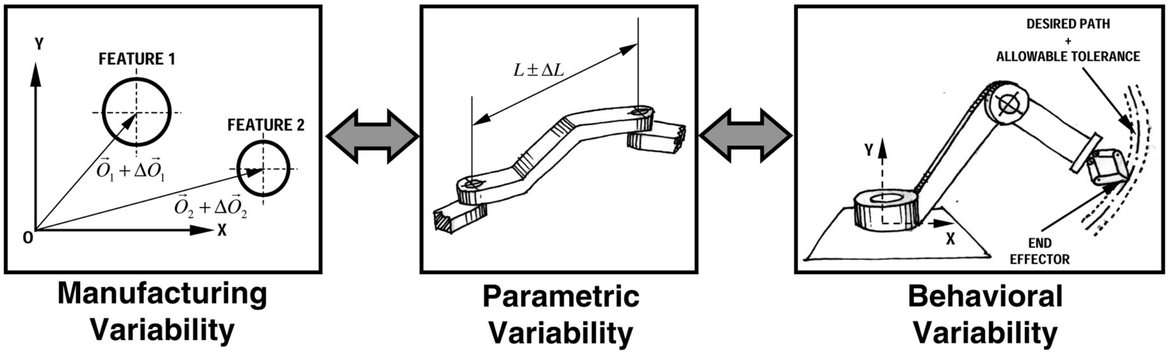

Figure 2. RELATING BEHAVIORAL SPECIFICATIONS TO MANUFACTURING TOLERANCES

ative "feature blindness." Moreover, tolerances that specify mate and fit quality in the traditional sense are not suitable for *in-situ* fabrication of assemblies, as gaps and clearances manifest themselves directly in the geometry of the support structure and there are no assemblability issues that can arise (Rajagopalan and Cutkosky, 1998). Finally, SFF is performed, in most cases, using a series of process steps (not unlike VLSI fabrication). Thus the achievable end-result accuracy is dependent on many factors and is not as easily describable as the precision of a machine tool.

In this paper, we assume that the positional accuracy with which a joint in a mechanism can be located can be characterized as a tolerance region around the joints' neighborhood. In the most general case this region can have arbitrary geometry (anisotropic and non-homogeneous) depending upon the characteristics of the process workspace.

Behavior of Non-Ideal Mechanisms

Before discussing the non-ideal behavior of mechanisms, it is important to fully and unambiguously define their desired *ideal* behavior. Fabrication processes are inherently inaccurate, necessarily resulting in non-ideal parts. Errors in manufactured geometry could have a multitude of contributing factors - machine-tool precision, workpiece compliance, material properties, thermal stresses, etc. However, it is important to realize that not all manufacturing errors need contribute towards degrading a mechanism's *intended* behavior. Researchers have developed formal languages by which designers can specify mechanical components in terms of intended behavior rather than by limits on geometric variability (Joskowicz and Neville, 1996). Eventually however, manufacturing processes can only be concerned with parameters that are observable and controllable during fabrication. Thus, it is important that any behavioral specifications be eventually translated to geometric tolerances on the measured dimensions of the workpiece. Examples of behavioral specifications on mechanisms are desired end-effector accuracy at a finite number of positions, nominal path to be followed by an end-effector along with maximum allowable variation from the path-profile, higher-order (i.e. velocity/acceleration) properties at a design point, etc.

The relationship between behavioral specification and manufacturing tolerances can be studied in two stages. The first is mapping of behavioral specifications to specifications on the variability of the mechanism configuration parameters (i.e. link lengths, joint angles etc.). The second is the mapping of parametric variability to the locational accuracy of the manufacturing environment. Figure 2 illustrates this concept. We hope to address the first problem (i.e. mapping the desired behavior of the mechanism to limits on link parameters) with on-going and future research. In this paper, we make the simplifying assumption that the manufacturing variability has already been characterized and the objective is to find the configuration for which the worst-case variability of the geometric parameters (i.e. link lengths) is minimized.

Computational Geometry: Definitions and Assumptions

In this section, we define some of the entities and concepts that are important in the later treatment of the problem.

Compact and Simply-Connected: We make the assumption that the regions under consideration are compact (i.e. closed and bounded) and simply connected. This assumption is realistic for most engineering applications, and will eliminate many special cases in our analysis. From this point on, any reference to a set/space/region will imply compactness and simply-connectedness, unless otherwise specified.

Distances: Mathematicians have developed a generalized notion of *distance* (Blumenthal, 1953). Closely related to distance are *position* and *size*, both of which are concepts central to our discussion. The most common understanding of distance is in the Euclidean sense, where one is concerned with the shortest path between two geometric entities (e.g. points) in a Euclidean n-space. A function δ defined on two elements a and b of a metric space $\mathcal{M}$ is called a *distance* iff $\delta(a, b)$ is *positive, symmetric,* and obeys the *triangle inequality*. In this treatment we are concerned with the Euclidean distance metric, which is given by the Pythagorean Theorem.

There are two approaches to determining the distance between geometric entities like points, curves and regions (surfaces and volumes) – analytical and computational. Given an analytic description of the entities, it may be possible to compute the extremal distances between the entities in closed form. However, the problem of computing raw distances between these entities is analytically tractable only for the simplest of cases (e.g. points, straight lines, simple conics) due to the highly non-linear algebraic equations generated (for example, computing the minimum distance between two non-intersecting spatial cubics is a non-trivial problem analytically). An alternative approach is to discretize the entities (into finite point-sets) within some region of interest, and to arrive at the solution by executing an efficient search algorithm. This approach works for most practical applications, and is especially appropriate when the geometry has special properties like being convex, disjoint and polygonal. This has been the preferred approach in robotics and in pattern recognition, and computational geometers have mostly concentrated on developing algorithms for determining distances between discrete point sets and polygonal regions. As noted earlier, there has been extensive research in the area of algorithms for computing distance properties, along with complexity and performance evaluation for various algorithms (Huttenlocher *et al*, 1994)(Toussaint, 1984)(Bhattacharya *et al*, 1983). The main difference between the approaches in these areas and that presented here is that they are concerned with intersections and Minkowski Sums of regions (for collision detection and navigation in robot motion planning), and Hausdorff distances between point-sets (for pattern matching). We are interested in extremal distances between disjoint tolerance regions, which have their relative motion restricted by kinematic constraints.

To keep the applicability of the approach as general as possible, we pose the problem assuming analytical descriptions of tolerance regions about the nominal joint positions. However, for tractability, we discretize the problem (or assume simple geometries), and look at efficient computational methods to solve it.

Extremal Distances Between Regions: The extremal distances between any two regions in space are the minimum and maximum Euclidean distances (denoted by D_{max} and D_{min}) between any two points in the spaces. i.e., if $\mathcal{R}_1$ and $\mathcal{R}_2$ represent two regions in Euclidean space, and p_i is a point in $\mathcal{R}_1$, q_j is a point in $\mathcal{R}_2$, then:

$$\begin{aligned} D_{max}(\mathcal{R}_1, \mathcal{R}_2) &= \max[\delta(p_i, q_j)] \\ D_{min}(\mathcal{R}_1, \mathcal{R}_2) &= \min[\delta(p_i, q_j)] \\ &\forall p_i \in \mathcal{R}_1, q_j \in \mathcal{R}_2 \end{aligned} \tag{1}$$

Researchers in pattern matching have mostly been concerned with determining efficient algorithms for optimizing a distance metric (typically minimizing the Hausdorff distance (Huttenlocher *et al*, 1994)) between point-sets with a finite number of points. Efficient algorithms for determining the minimum and maximum Euclidean distances between point-sets have also been developed, along with the associated complexity analysis.

Minimum Distance between Point-Sets: If $\mathcal{S}_1 = \{p_1, p_2, \cdots, p_n\}$ and $\mathcal{S}_2 = \{q_1, q_2, \cdots, q_n\}$ represent two sets of points on a plane, then the minimum distance between the two sets is given as:

$$D_{min}(\mathcal{S}_1, \mathcal{S}_2) = \overset{\min}{i, j}\, [\delta(p_i, q_j)]; i, j = 1, 2, \cdots, n \tag{2}$$

The naive strategy of examining the distance between every point-pair (p_i, q_j) results in $O(n^2)$ complexity. Algorithms that run at $O(n \log n)$ worst case complexity have been demonstrated (Toussaint, 1984). It has been shown that the lower-bound on the complexity of this problem is $\Omega(n \log n)$. The optimal algorithms work by computing the Minimum Spanning Tree (MST) of the union of the two given sets (i.e. $\mathcal{S}_1 \cup \mathcal{S}_2$). The minimum distance is an edge of the MST.

Maximum Distance between Point-Sets: If $\mathcal{S}_1 = \{p_1, p_2, \cdots, p_n\}$ and $\mathcal{S}_2 = \{q_1, q_2, \cdots, q_n\}$ represent two sets of points on a plane, then the maximum distance between the two sets is given as:

$$D_{max}(\mathcal{S}_1, \mathcal{S}_2) = \overset{\max}{i, j}\, [\delta(p_i, q_j)]; i, j = 1, 2, \cdots, n \tag{3}$$

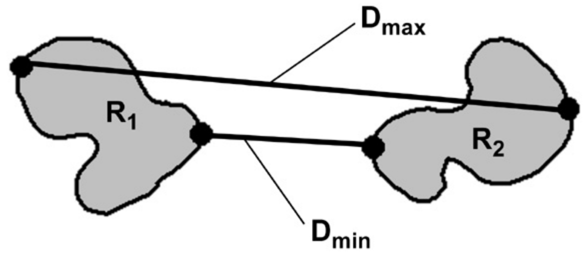

Figure 3. EXTREMAL DISTANCES BETWEEN TWO PLANAR TOLERANCE REGIONS

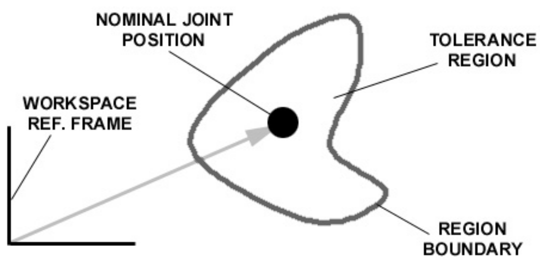

Figure 4. BOUNDARY OF A TOLERANCE REGION

Once again, the naive strategy of comparing the distance between every point pair results in $O(n^2)$ complexity. Optimal algorithms that run in $O(n \log n)$ worst case complexity, which are within a constant factor of the theoretical lower bound of complexity, have been demonstrated (Bhattacharya *et al*, 1983). The algorithms work by proving that the maximum distance is supported by points on the convex-hulls of the two regions.

Boundary of a Planar Region: The *boundary* $B(\mathcal{R})$ of a set (or region in space) $\mathcal{R}$ is the set of all points that belong to the closure of the set $\mathcal{R}$ as well as the closure of the complementary set $\bar{\mathcal{R}}$. The boundary is also sometimes called the *frontier* of the region. The boundary of a compact and simply-connected planar region is a non-self-intersecting closed planar curve. The *sense* or *orientation* of the curve is defined such that traversal of the curve along its positive sense puts the interior of the region on the RHS. When defining tolerance regions, the interior of the region contains the nominal position of the toleranced feature.

Boundary Points and Internal Points: A *boundary point* is a point on the boundary of a set or region. It has the property that its neighborhood has at least one point within the set, and at least one outside of it. An *internal point* is any point in the set that is not a boundary point. All points in the neighborhood of an internal point lie within the set.

Computational Geometry: Theorems

Here, we state some relevant theorems without the associated proofs. Proofs of these theorems are either covered by texts and research papers in computational geometry (Preparata, 1985), pattern matching and robot motion planning (Latombe, 1991), or are simple extensions of the known concepts.

Theorem 1: The extremal distances between two compact and simply-connected regions in Euclidean space are supported by points on the boundary of the regions.

Lemma 1.1: The minimum distance between two convex, disjoint, and polygonal regions is either supported by vertices of the polygons, is an orthogonal distance between a vertex and an edge of the polygons, or is an orthogonal distance between two edges of the polygons.

Lemma 1.2: The maximum distance between two convex, disjoint and polygonal regions is supported by vertices of the two polygons.

Theorem 2: Given two finite point sets, the minimum distance between them is supported by an edge of the *minimum spanning tree* of the union of the two sets.

Theorem 3: Given two finite point-sets, the maximum distance between them is supported by points on the *convex hull* of the two sets.

These theorems imply that, in general, it is adequate to examine the boundaries of regions to determine extremal distances between them. Furthermore, for some special regions (like convex polygonal regions and discrete point-sets) it is adequate to examine just a subset of the boundary itself.

PROBLEM FORMULATION

As mentioned earlier, our approach is to cast the problem of finding the optimal build pose as that of locating the configuration of the mechanism for which the worst-case variation of the link sizes is at a minimum. We are given regions of arbitrary geometry in the neighborhood of the *nominal* joint positions, inside which the actual joint is assumed to lie. These regions are assumed to be dense, compact (i.e. closed and bounded) and simply-connected. Each point within the region is assumed equi-probably be a candidate for the actual joint position. The shape of the tolerance region around each nominal joint position is a function of the workspace within which the mechanism is being built. In the most general case, the workspace is

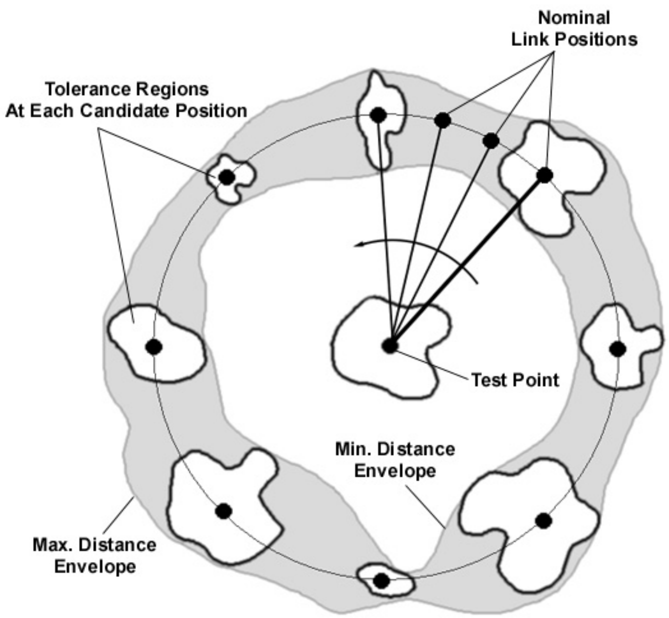

Figure 6. TOLERANCE REGION MOVING IN A EUCLIDEAN PLANE

both non-homogeneous and anisotropic (i.e. the tolerance region varies both as a function of absolute position and of direction).

The Single Link

In this section we look at the problem of finding the optimal build position for a single isolated link. If the desired objective is to maintain the resultant link-length as close as possible to the nominal link length, the the problem is identical to finding the orientation (relative to a workspace datum frame) for which the difference between the two extremal distances (i.e. max and min distances) is a minimum. The problem can be solved by examining the extremal distances under constrained relative motion of the nominal joint positions (and consequently, the relative motion of the tolerance regions for the nominal positions).

As we did in the earlier section on Definitions, let us denote the maximum and minimum distances between two tolerance regions $\mathcal{R}_1$ and $\mathcal{R}_2$ by $D_{max}(\mathcal{R}_1, \mathcal{R}_2)$ and $D_{min}(\mathcal{R}_1, \mathcal{R}_2)$ respectively. These distances can either be determined by discretizing the boundary of the regions, or by examining the vertices and edges (for convex, polygonal regions), as appropriate.

Any relative Euclidean motion in the plane for the two regions can be thought of as a combination of a rotation and a translation. Thus, without any loss of generality, we can attain all relative positions by allowing region $\mathcal{R}_1$ to purely rotate, and region $\mathcal{R}_2$ to purely translate with respect to a local reference frame. In the case of a revolute joint, however, the nominal positions of the two regions stay at a fixed distance with respect to each other. Thus, it is adequate to let one region purely rotate about the origin, while the other remains stationary with respect to the local frame. The origin of the local frame can itself translate anywhere in the fixed reference frame, and the regions can, in general, transform shape as they navigate the workspace (this could happen, for example, in a compliant robotic manipulator that inserts joints with different locational accuracy at various points in its work envelope). The nature of these transforms will depend upon the workspace characteristics, but they cannot affect the nominal position of the joints. The optimization problem can now be formulated as follows:

Find the orientation $\{\theta^\star, x^\star\}$ for the link such that the minimum variability, defined as:

$$\Delta_{min}(\mathcal{R}_1, \mathcal{R}_2) = \min_{\theta, x}\{D_{max}[s_{\theta x}(\mathcal{R}_1), g_{\theta x}(\mathcal{R}_2)] - D_{min}[s_{\theta x}(\mathcal{R}_1), g_{\theta x}(\mathcal{R}_2)]\} \quad (4)$$

is achieved, where θ and x are parameters that denote the orientation of the link in the local frame, and the absolute position of the local frame, and $s_{\theta x}$ and $g_{\theta x}$ are general transformations that operate on tolerance regions. Examples of possible transforms are linear, affine transforms (scale, rotate etc.), or non-linear mappings that characterize the workspace of a compliant robot. For the special case where the shape of the tolerance region attached to a joint location is independent of its absolute position in the workspace, equation 4 becomes:

$$\Delta_{min}(\mathcal{R}_1, \mathcal{R}_2) = \min_{\theta}\{D_{max}[S_\theta \cdot \mathcal{R}_1, \mathcal{R}_2] - D_{min}[S_\theta \cdot \mathcal{R}_1, \mathcal{R}_2]\} \quad (5)$$

where $S_\theta \cdot \mathcal{R}_1 = \{S_\theta r_i \mid r_i \in \mathcal{R}_1\}$ is a rigid body rotation matrix applied to $\mathcal{R}_1$.

Since $\Delta_{min} = D_{min} - D_{max}$ can be calculated at each position in $O(n \log n)$ time, the complexity of the optimization problem is $O(C^3 n \log n)$, where C depends upon the resolution with which θ and x are discretized. The cubic term in the complexity is on account of the three degrees of freedom of the link - position and orientation on a plane.

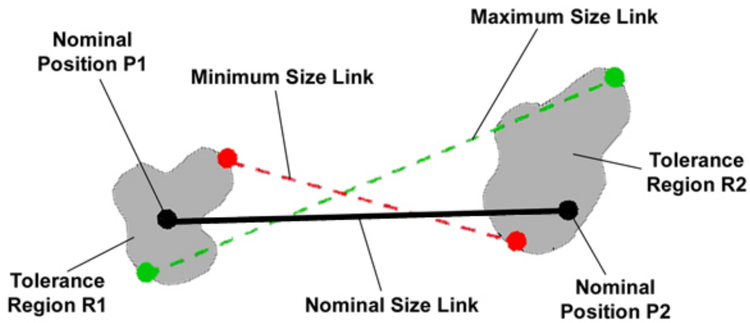

Figure 5. TOLERANCE REGIONS REPRESENTING LINK LENGTH VARIABILITY FOR A SINGLE LINK

The Multi-Link Mechanism

The multi-link problem is a simple extension of the single link problem, if the objective is still to keep the resultant error in link-lengths as small as possible. In this case, the single link problem is successively encountered at each nominal joint position, with regards to the next adjacent link in the mechanism. If there are m links and k degrees of freedom for the mechanism, we want to determine the mechanism configuration $\{\theta_1^\star, \theta_2^\star, \theta_3^\star, \ldots, \theta_{k-2}^\star, x^\star\}$, such that the minimum cumulative variability, defined as:

$$\Delta_{min}^T = \overset{\min}{\theta_1, \theta_2, \theta_3, \ldots, \theta_{k-2}, x}\{\sum_{i=1}^{m-1} \Delta_{min}(\mathcal{R}_i, \mathcal{R}_{i+1})\} \qquad (6)$$

is attained, where Δ_{min} is obtained as in the previous section.

Note that this is the configuration for which the global minimum of the cumulative variability of all links is attained. We call this the *optimal pose*. The complexity of the search is $O(mC^k n \log n)$.

Practical Considerations

The technique for finding the optimal pose outlined above has no awareness of the many practical constraints that may limit the domain of available choices for mechanism poses. An obvious example of such a limitation is that joints in the same plane cannot overlap, and links in the same plane cannot cross each other. Another possible constraint is that mechanisms cannot stretch outside the available machine workspace (in general, smaller mechanism footprints are preferable). In addition to these workspace constraints, the program has no kinematic knowledge (e.g. branching). For example, a closed-link mechanism built in the optimal pose may not be able to return to every working configuration without dis-assembly of the joints.

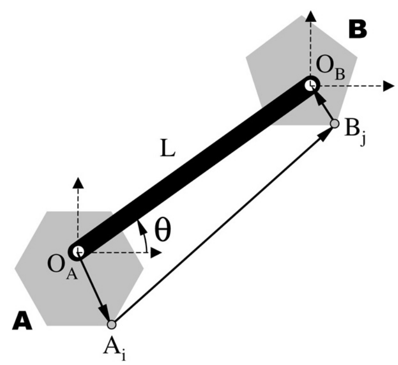

Figure 7. LOOP EQUATIONS FOR A SINGLE LINK WITH CONVEX TOLERANCE REGIONS

It is necessary for the optimizing algorithm to take such constraints and preferences into account in order to produce a viable set of optimal mechanism poses. Non-overlap and non-crossing constraints are easily encoded in the solver as restrictions on the relative orientations of links (either adjacent links or all links), for example:

$$\theta_{min} \leq \|\theta_i - \theta_j\| \leq \theta_{max} \qquad (7)$$

Other constraints (e.g. kinematic constraints) are more complicated to encode mathematically - but can be eliminated by inspection after the fact.

EXAMPLES

Here, we present some simple examples that serve to illustrate the concepts presented in the earlier sections. These examples were solved using spreadsheet simulation (as described in the following section). Engines that programmatically solve for the optimal pose given arbitrary tolerance geometries are under development.

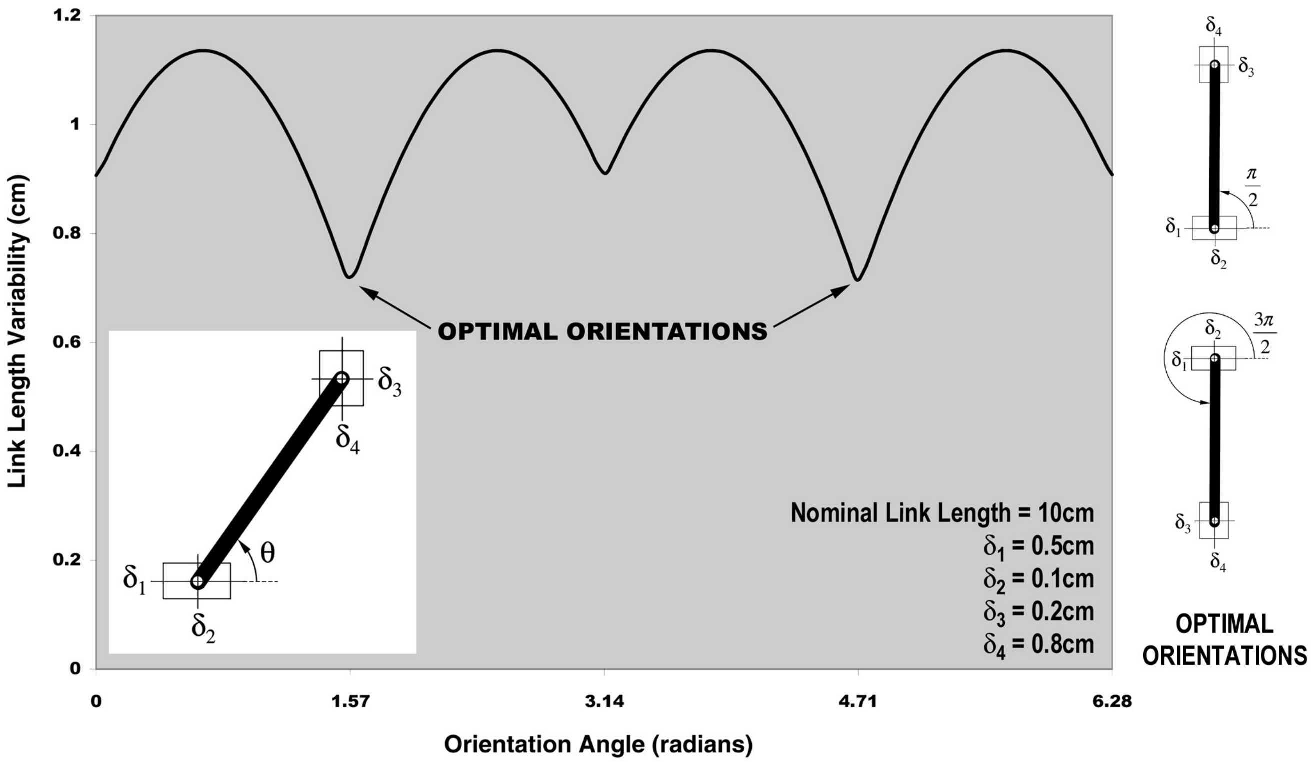

Figure 9. OPTIMAL ORIENTATIONS FOR A SINGLE LINK WITH RECTANGULAR TOLERANCE REGIONS

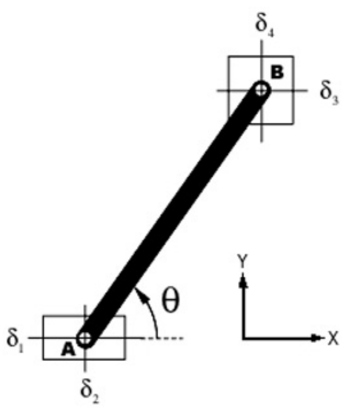

Figure 8. A SINGLE LINK WITH RECTANGULAR TOLERANCE REGIONS

Single Link

For convex, polygonal tolerance regions, the analysis is greatly simplified. The problem can be solved by writing pairwise loop-closure equations on boundary points on the two tolerance regions. For computing the minimum link length, vertices and points on the polygon edges need to be tested. However, for maximum link length, it is adequate to test the vertices of the polygons. The loop-closure procedure is illustrated in Figure 7. It yields a vector equation of the form:

$$Z_{O_A A_i} + Z_{A_i B_j} + Z_{B_j O_B} = Z_{O_A O_B} \tag{8}$$

which can be algebraically solved for the unknown scalar length $\|Z_{A_i B_j}\|$. The simulation proceeds by solving for the maximum and minimum link lengths at each step in the link orientation.

To illustrate this method, we have used the case of rectangular tolerance regions around the nominal position of the joints. Figure 8 shows a diagram of this problem. This would correspond to the case where the worst-case positional accuracy of the process is expressed in the plus/minus sense for the two axes on the working plane. The characteristic values used in this example have been picked based only upon a superficial characterization of the SDM process. Characterizing the achievable location tolerances for SFF processes is an important issue that we hope to address in future stages of the research. We have also assumed that the tolerance regions for the two joint positions are different (this could happen, for example, when the position of

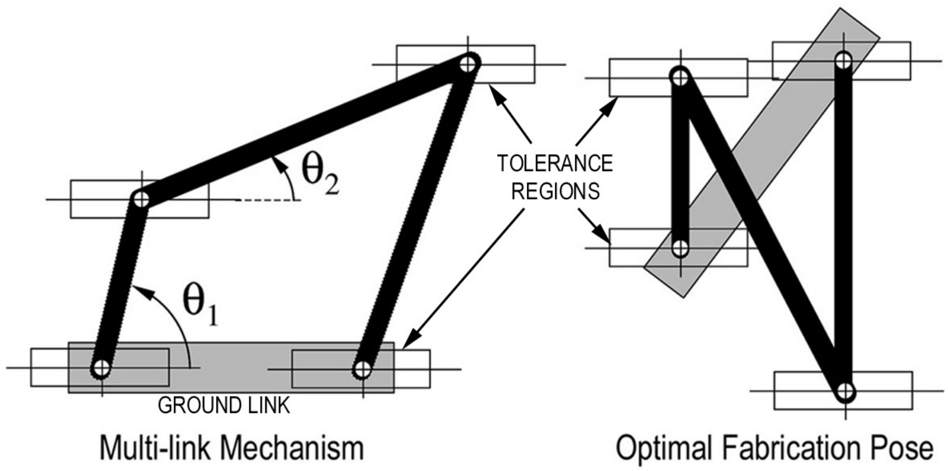

Figure 10. OPTIMAL ORIENTATION FOR A 4-BAR MECHANISM WITH RECTANGULAR TOLERANCE REGIONS

the first-built joint is known to a higher accuracy than that of the second-built joint due to some in-process measurements), and that they are indifferent to the absolute position of the link in the workspace. Figure 9 shows a plot of the variability in the link length as a function of the orientation of the link in the workspace. The *link length variability* plotted on the Y-axis is simply the difference between the maximum and minimum possible link lengths at each candidate orientation angle (θ). The optimal orientations (indicated) are those for which the link-length variability is at a minimum.

Multi-link Mechanism

Here, we consider the example of a four-bar mechanism. The figure (Figure 10) shows the mechanism with the rectangular tolerance regions associated with each joint. Also shown is the resultant optimal orientation, after the practical constraints have been taken into account. We again assume that the shapes of the tolerance regions are independent of the absolute position within the workspace, hence there are two degrees of freedom (i.e. two orientations) associated with this mechanism. Given the shape of the tolerance regions (elongated along the X-axis), one would intuitively expect the optimal pose to be fully folded along the Y-direction. However, the physical constraints (e.g. link overlap) would not allow that configuration, and the illustrated pose implements a minimum link angle constraint between adjacent links.

CONCLUSIONS AND FUTURE WORK

In this paper, we have demonstrated a method for reasoning about the optimal manufacturing configuration of mechanisms slated for fabrication by layered manufacturing, given certain assumptions about process capability and desired behavioral objective for the mechanism. This is one portion of an ongoing effort at Stanford University and other partner institutions to develop software and hardware tools that make SFF processes more useful to designers of electro-mechanical systems. As these methods and tools mature, it will be possible for designers of electromechanical systems to design and fabricate complex, integrated devices with embedded electronics and precise mating and fitting components.

An important issue that we hope to address with ongoing research is that of servicing multiple design objectives using the techniques described in this paper. For example, we would like to answer the following question - “What is the optimal build pose for a four bar mechanism that has to pass through four design points in its workspace with high accuracy?” Other design objectives could include accurate path-following by the end effector of a open-chain mechanism and higher-order (velocity, acceleration) characteristics of the mechanism at some design points. We also hope to extend this approach to general spatial mechanisms by formulating the problem as that of positional uncertainty between spatial screws that represent the joint constraints.

Clearly, another important issue that needs to be addressed is that of characterizing the process capability of existing manufacturing processes, in terms of achievable location and form tolerances. This too is part of ongoing research at Stanford and other institutions that are developing SFF technology. As the capability to quickly prototype functional devices becomes an essential component in the product design cycle, it is hoped that such process characterization will become customary in the commercial RP industry.

ACKNOWLEDGMENT

The authors gratefully acknowledge the support by the National Science Foundation under NSF-MIP9617994. We also wish to thank Jorge Cham and Mike Binnard (for use of mechanisms built by them as examples), Maria Yang (for her assistance with proof-reading and for offering helpful suggestions) Prof. Fritz Prinz and all other students and staff at the Center for Design Research and the Rapid Prototyping Laboratory at Stanford University.

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