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TOLERANCE REPRESENTATION FOR MECHANISM ASSEMBLIES IN LAYERED MANUFACTURING

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ABSTRACT

By building parts up incrementally in layers or "compacts," 3D layered manufacturing processes are capable of fabricating parts with complex internal geometry and multiple materials. Additionally, since parts are built fully embedded in a sacrificial support material, it is possible to fabricate fully assembled devices with multiple moving parts and mating/fitting surfaces (such as mechanisms). Given the method of manufacture, it becomes possible to provide designers of linkages with some flexibility in expressing constraints on the manufactured geometry of these parts. For example, designers now have the option of directly specifying the geometry of gaps and clearances with explicit tolerance assignments. In processes like Shape Deposition Manufacturing (SDM), these features are directly manifest in the geometry of the support structure. In many cases, it is these "shadow" features that most directly govern the behavior of the artifact being built. This paper explores the impact of *in-situ* manufacturing of assembled mechanisms on the tolerances required. Relaxation of the assemblability constraint is shown to result in looser manufacturing tolerances and better control over gap geometry for the specific instance of an isolated pin-joint.

KEYWORDS: Tolerance Representation, Solid Freeform Fabrication, Layered Manufacturing, Kinematic Joints, Assembly.

NOMENCLATURE

D : Diameter of thrust plate
L : Depth of hole
d : Diameter of shaft
l_{CA} : Distance from shaft center to task location
Δ_a : Axial clearance
Δ_d : Diametrical clearance
$\Delta_{desired}$: Desired bearing clearance
$\Delta_{\max}$: Maximum as - manufactured bearing clearance
$\Delta_{\min}$: Minimum as - manufactured bearing clearance
$\delta_{\max}$: Upper bound on positional error
$\varepsilon_{\text{H-size}}$: Best achievable hole size tolerance
$\varepsilon_{\text{H-location}}$: Required position tolerace of hole at MMC
$\varepsilon_{\text{S-size}}$: Best achievable shaft size tolerance
$\varepsilon_{\text{S-location}}$: Best achievable shaft position tolerance
$\varepsilon_{\text{C-size}}$: Best achievable circular feature size tolerance
$\phi_{\text{H-nom}}$: Nominal hole diameter
$\phi_{\text{H-min}}$: Maximum material size of hole
$\phi_{\text{S-nom}}$: Nominal shaft diameter
$\phi_{\text{S-max}}$: Maximum material size of shaft
$\phi_{\text{S-actual}}$: Actual (measured and fitted) shaft diameter

INTRODUCTION

A fundamental achievement of manufacturing in the antebellum period (1820-1860) of American history was the technology to fabricate interchangeable parts. Interchangeability, which was originally advocated by the French General Jean-Baptiste de Gribeauval as early as 1765 [Hounshell, 1984] was first realized on a large scale by the arms contractors Simeon North and Eli Whitney. The driving force was the strategic advantages offered by the ability to exchange faulty musket and pistol parts with replacements recovered from other faulty arms on the battlefield. The intellectual leap that took place during this era was the conceptualization of the mechanical ideal. This ideal represented perfection – usually in the geometric sense – and every physical artifact actually built was but an imperfect copy. Methods were devised that mapped behavioral restrictions to constraints on part geometry. Functional (go, no-go) gages and process call-outs implicitly represented limits on the permissible variability from the mechanical ideal.

Other supporting technologies that made it possible to realize part interchangeability included advancements in model-making, jigs and fixturing, and metrology. Dimensioned drawings with explicit tolerances eventually replaced representational drafting practices. The first drafting and tolerancing standards were articulated in the 1930's and still continue to be refined. These major advancements in manufacturing, combined with the advent of mechanization, enabled the mass-scale production of nearly identical parts that we see to this day. A vast majority of common mass-produced devices consist of assemblies of parts that are individually and independently fabricated.

Much of modern manufacturing continues to be dependent on (and benefit from) the capability to fabricate interchangeable parts which assemble freely with each other. However, some recent technologies make it possible to manufacture entire "assemblies" incrementally, with no subsequent assembly operations required. In these methods of manufacture (e.g. Stereolithography [3D Systems]; Shape Deposition Manufacturing [Merz, 1994]; Selective Laser Sintering [Bourell et al, 1990]; Laminated Object Manufacturing [Helysis, Inc]; 3D Printing [Sachs 1992]; etc.), solid 3D mechanical parts are built by using a combination of material addition and removal processes. Many of these processes work with planar 2.5D layers, while others (like SDM) operate with full 3D layers. In order to construct parts, an electronic (3D CAD) representation is first decomposed into simpler primitives – layers or "compacts" [Merz et al, 1994] – and process-plans and tool paths are generated subsequently for each individual primitive. Optionally, metrological tools like CMMs or touch-probes can be used to gather data about the artifact being built during the process.

Figure 1 compares the manufacturing model employed in layered manufacturing as opposed to conventional manufacturing techniques. There are several benefits associated with layered manufacturing. Most importantly, decomposition significantly reduces the planning complexity associated with global feature interactions, which have been the bane of traditional CNC manufacturing [Gadh and Prinz, 1995]. Layered manufacturing allows physical access to the entire volume of the part (not just the surfaces), thus making possible novel designs involving multiple materials, complex internal geometries or embedded components [Fessler et al, 1996]. Additionally, these processes build parts fully encapsulated in a support structure, which makes it possible to fabricate multiple-part assemblies with bearing, mating and fitting surfaces *in-situ*. An example of such a part, built at the Stanford Rapid Prototyping Laboratory, is shown in (Fig. 2). Many of the assemblies built in this manner cannot be disassembled without breakage, and typically, there is no possibility of interchanging their parts with other similar mechanisms. This feature of layered manufacturing is particularly relevant in the context of tolerance representation.

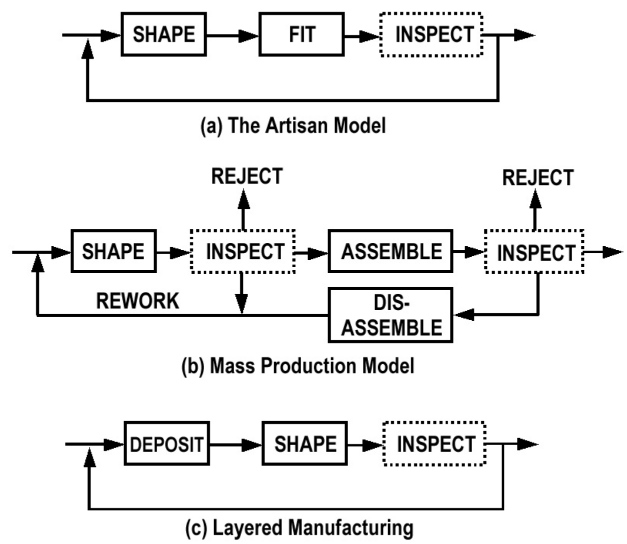
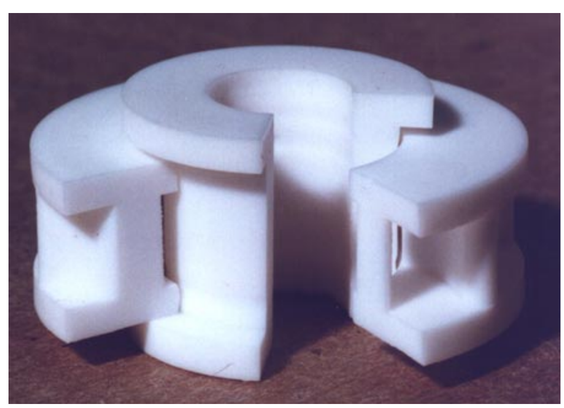


Fig. 1: Models of manufacturing practice

In this paper, the issue of tolerance assignment and representation for designs that are slated for manufacture by a layered process is explored. In particular, tolerances on gaps and clearances between bearing surfaces, which subsequently determine the kinematic and dynamic behavior of mechanisms, are addressed. The example of a simple pin-joint is used as an illustration, and an alternate tolerance representation for the bearing is proposed.



Courtesy: Alexander Cooper, Rapid Prototyping Lab, Stanford

Fig. 2: Cutaway of a multi-part assembly built *in-situ*

BACKGROUND REVIEW

It has been noted that the advent of two relatively recent phenomena, namely Computer Aided Design (CAD) and Co-ordinate Measuring Machine (CMM) technology, have precipitated a rash of interest in the field of tolerance representation [Voelker, 1993]. Much of the recent theoretical work in tolerancing has concentrated on one of two important problems – the mathematization of Geometric Dimensions and Tolerances (GD&T) (e.g. [Walker and Srinivasan, 1993]), and the development of a "rational basis" for tolerance assignment based upon the functional requirements of a part [Srinivasan and Wood, 1995]. A fair amount of thought has also gone into determining the most appropriate mechanism of associating tolerances with solid geometry. This includes representations that are CSG based [Requicha, 1984], [Requicha and Chan, 1986], Virtual Boundary Requirements based [Jayaraman and Srinivasan, 1989] or Hybrid CSG/B-rep and feature based [Turner, 1987], [Roy and Liu, 1988]. Researchers have also developed data structures which make it easy to reason about over-toleranced designs and improve the efficiency of tolerance analysis on evolving designs [Tsai and Cutkosky, 1997]. However, few commercial CAD systems support feature-based geometric dimensioning and tolerancing. The latest release of AutoCAD®, for example, treats tolerances as annotations in the drafting mode. Tolerances are not directly associated with specific topological entities, making it difficult to retain (or automatically translate) the tolerance information when the model is decomposed or otherwise modified. Since layered manufacturing is closely tied to CAD (it cannot operate in the absence of an electronic representation), it suffers in the presence of any deficiencies in state-of-the-art geometric/CAD engines.

An extensive survey of data-analysis algorithms and CMM technology conducted by the Government-Industry Data Exchange Program in 1988 found many serious problems with the state-of-the-art [Walker, 1988], [Feng and Hopp, 1991]. Since then, a fair amount of work has been done in the area of geometric approximation of surface sample measurements and robust data-analysis algorithms. Of particular interest is the development of Chebychev fitting algorithms for manufacturing features like planes, cylinders, cones and spheres [Chaterjee and Roth, 1997], which apply to bearings with cylindrical or spherical (ball-and-socket) surfaces.

To this day, much of tolerancing in the manufacturing industry is seen as an art – ad-hoc assignments based upon the experiences of individual designers and fabricators, or traditional tolerancing practice captured in design handbooks. Recently, however, tools like statistical and geometric tolerancing have increased the expressiveness of tolerancing while simultaneously reducing the precision required from the manufacturing facility in order to ensure functional parts. Statistical tolerancing exploits the variance limiting property of the overall dimension as independent and identically distributed (IID) tolerances stack up in a tolerance chain [Evans, 1975]. Geometric tolerances allow for tolerance zones with arbitrary geometry as opposed to the prismatic zones afforded by the conservative (worst-case) plus-minus tolerances [Requicha, 1983]. Criteria such as the Maximum Material Condition (MMC) and Least Material Condition (LMC) allow looser positional tolerances on features based upon size measurements at the inspection stage (a larger hole can drift more from its ideal position than a smaller hole – and still preserve assemblability). ANSI Y14.5M [ANSI, 1994] is a geometric dimensioning and tolerancing standard that includes specification of size, location, form, profile and run-out tolerances that has recently enjoyed wide use in industry.

It is acknowledged by researchers that surprisingly little is known about tolerancing for assemblies [Voelker, 1993]. In traditional engineering practice, assembly drawings are rarely toleranced. The LMC and MMC criteria, coupled with simple one-dimensional stack-up rules, ensure satisfactory assembly of individual parts. In bearing design, for example, given the type of fit and function (e.g. clearance fit for precision machinery), standard tables in mechanical design handbooks specify allowable size tolerances on the mating shaft and hole features. Representation of feature variations using homogeneous matrix transforms has been addressed by [Whitney and Gilbert, 1993]. This representation facilitates statistical tolerance stack-up and error propagation analysis in sequential assemblies. Assembly analysis of complex parts typically requires Monte Carlo simulation [Turner, 1993]. The focus of these methods is to predict the likelihood of successful assembly given allowable feature variations.

Any future exchange format for engineering designs should, at a minimum, include the geometric tolerance representation scheme outlined in the ANSI standards (e.g. the STEP standards [ISO, 1994]). However, the existing ANSI representation does not allow designers to express functional tolerances or allow designers of layered parts to specify tolerances in non-traditional ways that might be easier or cheaper to process. Some more convenient tolerance constructs are proposed in the following sections.

AN ALTERNATIVE TOLERANCE REPRESENTATION

With much of layered manufacturing, assemblability need not be an issue while assigning feature tolerances. As mentioned earlier, mechanisms can be simultaneously manufactured and assembled *in-situ*. Consequently, designers can specify tolerances dictated solely by behavioral constraints on the final artifact. Furthermore, in processes like Shape Deposition Manufacturing (SDM), geometry that is otherwise only indirectly specified (i.e. gaps, clearances etc) is directly manifest in the geometry of the support structure. This gives designers the option of exercising control over the behavior of the artifact by tolerancing these "shadow" features. Often, these features are the ones that most directly and substantially determine the behavior of a machine or mechanism. For example, the kinematic behavior of a pin-joint is typically most sensitive to errors in the designed geometry of the gap (i.e. the differential geometry between the hole and shaft), and not the specific geometry of the hole or shaft. A crucial step in this kind of tolerance assignment, therefore, becomes exploring the link between tolerances on gap geometry and predicted behavior for typical mechanical designs.

In order to fully exploit the advantages offered by layered processes, a data exchange format targeted for use by this community should incorporate constructs that allow for more flexible tolerance specification on assemblies which have functionally important gaps and clearances. Furthermore, for a certain class of well-understood kinematic fragments (like bearings), designers should be able to specify tolerances in a manner that captures the behavioral constraints while making the least commitment to a specific overall geometry. This will allow downstream manufacturing processes to infer or insert the appropriate geometry into the design based upon the specific manufacturing capability of the process. The following example builds one such construct for the specification of tolerances on pin-joints.

EXAMPLE – DESIGN OF A PIN JOINT

As an example, consider the design of a simple pin-joint. This type of joint is fairly common in mechanical design – especially in the design of linkages in robots and other machinery. Figure 3 shows the typical geometric parameters associated with a single pin-joint. Note that the first three parameters are regular feature sizes, while the last two are gap/clearance size parameters.

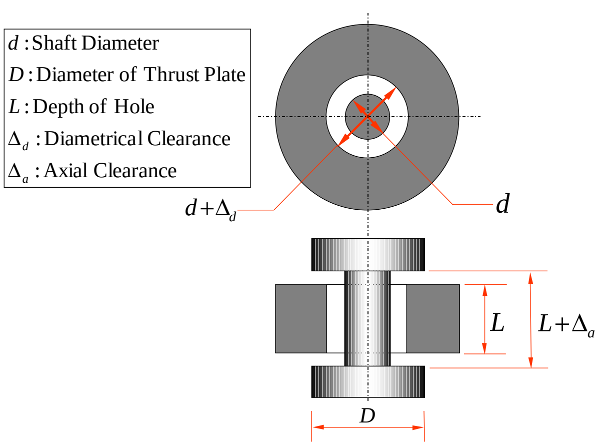


Fig. 3: A simple pin-joint

An ideal pin-joint would completely restrict all motion except rotational motion in the plane orthogonal to the joint axis. In a real pin-joint, however, the behavior of the joint depends upon the axial and diametrical clearances that result upon its manufacture (Fig. 4). Sometimes, these gaps are designed into the joint to provide for ease of assembly or hydrodynamic lubrication.

Researchers in applied kinematics [Wang and Roth, 1989] have addressed the problem of determining the maximum positional errors in mechanisms with non-ideal joints. Using their results, it is possible to compute the maximum allowable axial and diametrical clearances given some desired task tolerances and a contact configuration. (Fig. 5) shows one possible configuration (Mode 1-1 [Wang and Roth, 1989]) of an isolated pin-joint with some allowable tolerance ($\delta_{\max}$) at the task location A.

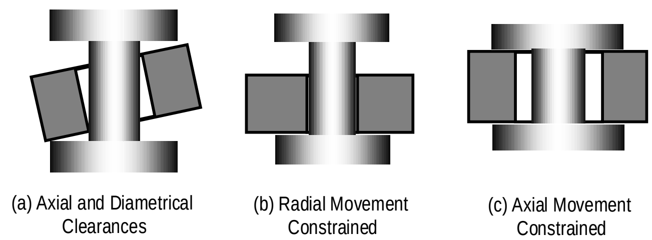


Fig. 4: Various axial and diametrical clearances

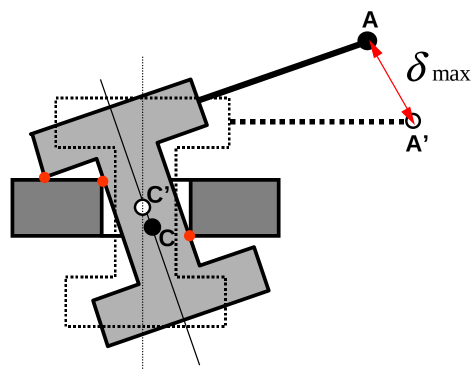


Fig. 5: A contact configuration for the pin-joint

(Fig. 6) shows an example single DOF crank mechanism in which this joint can be employed

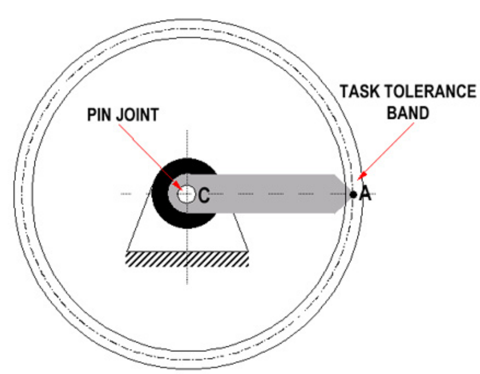


Fig. 6: Simple crank mechanism with pin-joint

The task tolerance band denotes the maximum acceptable deviation from the ideal (circular) path of the crank. In 3-space, the tolerance region is toroidal.

Wang and Roth's results can be re-written for the specific cases where the axis of constrained rotation (not the primary rotational axis of the pin) is taken orthogonal to the plane of the paper for all contact modes. This provides a simplified view of the maximum positional error at the task location based upon the pin geometry.

These results have been summarized for all possible contact modes in (Fig. 7), where:

A : Desired Task Location (a point in 3-space)

C : Geometric Center of the Shaft (a point in 3-space)

l_{CA} : Distance from Point A to C (scalar quantity)

$\delta_{\max}$: Upper Bound of Positional Error (scalar quantity)

Recall that Δ_a and Δ_d are defined in (Fig. 3). At zero positional error, the point C on the shaft is lined up with the center of the hole and the hole and shaft axes are collinear (dashed lines in Fig. 5). The zero error locations of points A and C are denoted by A' and C' respectively. The contact constraint for each mode needs to hold in order to maintain the illustrated contact configuration.

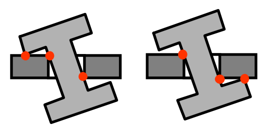
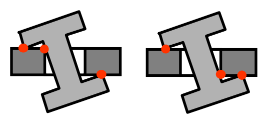
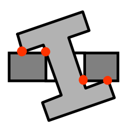
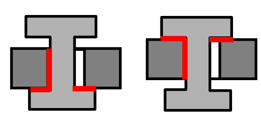
MODE	CONTACT CONFIGURATION	CONSTRAINT	MAX. POSITIONAL ERROR
1		$\frac{\Delta_a}{\Delta_d} > \frac{D}{L}$	$\delta_{\max} = \frac{\Delta_a}{2} + \left(\frac{2l_{CA} - d}{2L}\right)\Delta_d$
2		$\frac{\Delta_a}{\Delta_d} < \frac{D}{L}$	$\delta_{\max} = \left(\frac{2l_{CA} - L}{2D}\right)\Delta_a + \frac{\Delta_d}{2}$
3		$\frac{\Delta_a}{\Delta_d} = \frac{D}{L}$	$\delta_{\max} = \frac{l_{CA}}{D}\Delta_a$
4		–	$\delta_{\max} = \frac{1}{2}\sqrt{\Delta_a^2 + \Delta_d^2}$

Fig. 7: Max error for each contact configuration

MODE	Sensitivity of Task Error to Errors in Geometry				
	$\frac{\partial \delta_{\max}}{\partial D}$	$\frac{\partial \delta_{\max}}{\partial d}$	$\frac{\partial \delta_{\max}}{\partial L}$	$\frac{\partial \delta_{\max}}{\partial \Delta_a}$	$\frac{\partial \delta_{\max}}{\partial \Delta_d}$
1	0	$-\frac{\Delta_d}{2L}$	$-\frac{l_{CA}}{L^2}\Delta_d$	$\frac{1}{2}$	$\frac{2l_{CA} - d}{2L}$
2	$\frac{L - 2l_{CA}}{2D^2}\Delta_a$	0	$-\frac{\Delta_a}{2D}$	$\frac{2l_{CA} - L}{2D}$	$\frac{1}{2}$
3	$-\frac{l_{CA}}{D^2}\Delta_a$	0	0	$\frac{l_{CA}}{D}$	0
4	0	0	0	$\frac{\Delta_a}{2\sqrt{\Delta_a^2 + \Delta_d^2}}$	$\frac{\Delta_d}{2\sqrt{\Delta_a^2 + \Delta_d^2}}$

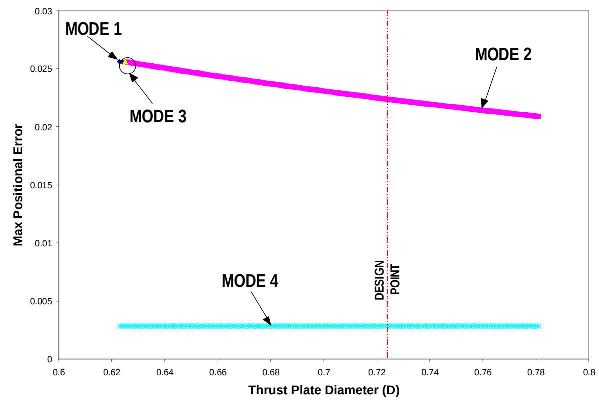
Fig. 8: Error sensitivity of task error

The equations in (Fig. 7) provide a basis for constraining the geometry of the pin-joint in order to assure that a certain task tolerance ($\delta_{\max}$) is achieved. Alternately, given the task tolerance (which depends upon the precision required for the task being performed), we can determine allowable values for axial and diametrical clearances based upon the geometry of the mechanism.

Figure 8 examines the sensitivity of the positional error to variations in the pin geometry, both for feature sizes and gap sizes. Two trends are identifiable from this table. First – an increase in feature size results in a decrease in positional error while an increase in gap size results in an increase in positional error. Second – given that gap sizes are typically much smaller than feature sizes (i.e. $\Delta_a, \Delta_d << D, d, L$), the value of the maximum positional error is much more sensitive to variations in gap sizes than to variations in feature sizes. In short, this analysis shows that, as expected, larger pins with smaller clearances are more accurate, and that *in manufacturing pin-joints, it is more important to control gap sizes than to control individual feature sizes.*

Pin Parameters : $D = 0.723", d = 0.623", L = 0.625",$
$l_{CA} = 4", \Delta_a = 0.004", \Delta_d = 0.004"$

MODE	Task Error Sensitivities for Example $\frac{\partial\delta_{\max}}{\partial D}$	$\frac{\partial\delta_{\max}}{\partial d}$	$\frac{\partial\delta_{\max}}{\partial L}$	$\frac{\partial\delta_{\max}}{\partial \Delta_a}$	$\frac{\partial\delta_{\max}}{\partial \Delta_d}$
1	0	−0.0032	−0.04	0.5	5.82
2	−0.0282	0	−0.0028	5.1	0.5
3	−0.0306	0	0	5.5	0
4	0	0	0	0.3536	0.3536



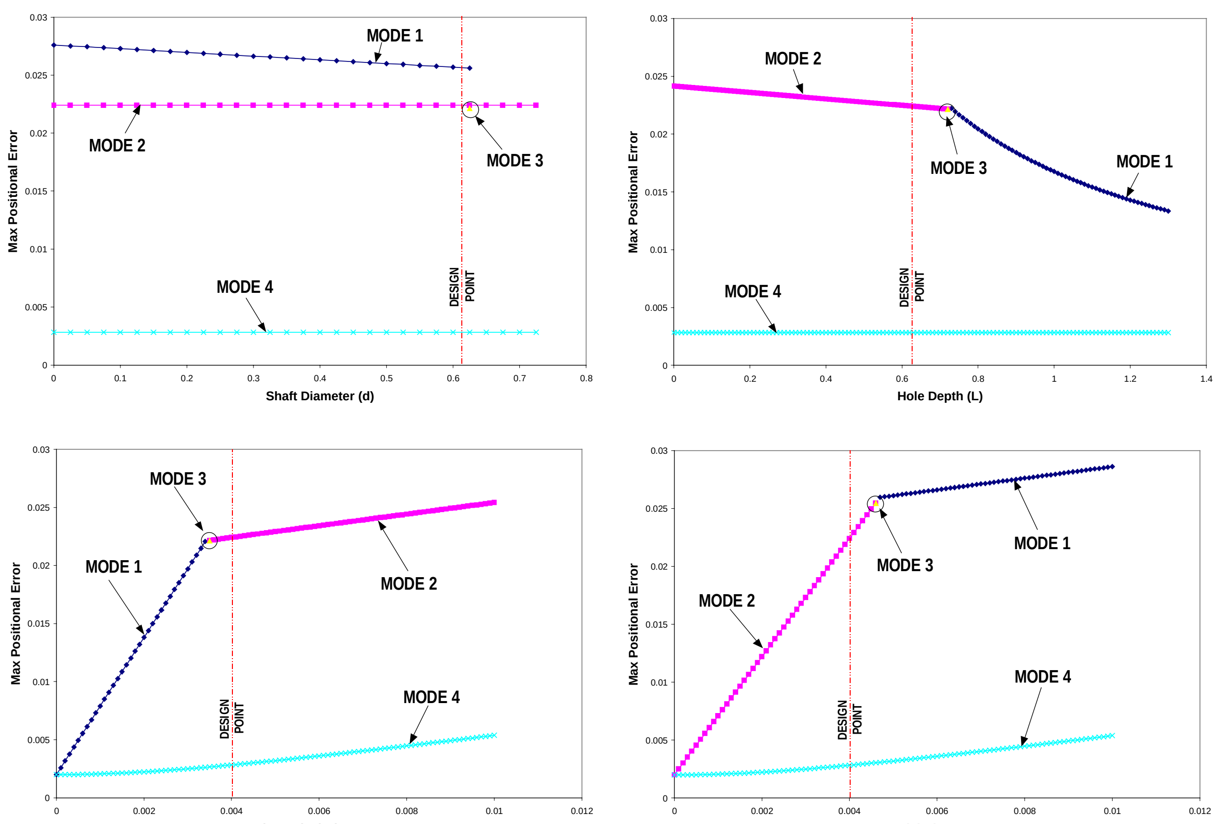


Fig. 9: Max positional error vs. pin parameter

The previous figure (Fig. 9) graphically illustrates how the task tolerance varies with pin geometry for a specific geometry (where the aspect ratios are typical for most applications). The particular values used in this example have been selected from a standard test specimen used to evaluate the fatigue life of a lubricated air-frame bearing [Thomas Register, 1997].

In each of the graphs, one of the five design parameters ($D, d, L, \Delta_a, \Delta_d$) is varied with the others held constant at the design point – until the contact condition is lost. Additional geometric and non-negativity constraints also apply (i.e. $D, d, L, \Delta_a, \Delta_d > 0; D > d$). The table on the top left shows the numeric values of the error sensitivities for this particular example. Note that the positional error computed is valid for any point that is at a distance l_{CA} from the geometric center of the pin.

The preceding section has demonstrated that for the typical (isolated) pin-joint, the kinematic behavior of the joint is more sensitive to errors in the designed geometry of the gaps than to errors in the size of the hole and shaft features.

In the following section, it is shown that layered manufacturing processes are able to control gap geometry more easily and accurately than traditional processes with sequential shape and assembly. An arbitrary axial clearance is assumed and the manufacturing precision required to obtain a certain diametrical clearance is derived with, and without, the assembly constraint.

ASSEMBLY AND NON-ASSEMBLY TOLERANCES

We assume that the following two parameters are given, and remain invariant in the two methodologies:

- The desired bearing clearance ($\Delta_{desired}$) – this will depend upon the behavioral requirements of the pin-joint (i.e. the desired fit-type and task tolerance).
- The best achievable tolerance on the size of the hole and shaft (ε_{H-size} and $\varepsilon_{S\text{-}size}$). These will depend upon the capability of a given process. Alternately, size-tolerance on both the circular hole and the circular shaft features could be assumed to be the same (ε_{C-size}).

We further assume that the worst-case form distortions (roundness, cylindricity etc.) are subsumed within the tolerance zone defined by the size tolerances. This is a valid assumption because size tolerances on hole/shaft features implicitly restrict maximum form tolerances (and similarly, position tolerances restrict size tolerances) [Requicha, 1983]. Form errors that remain within these bounds are assumed to not have an effect on joint behavior that is worse than that of the worst-case limiting material condition (LMC or MMC).

Method 1: Parts individually fabricated and assembled

This is the ANSI Y14.5M specified tolerancing method to ensure interference-free assembly. The parts that contain the hole and the shaft are assumed to be manufactured separately, using machines with similar capability.

We choose the nominal shaft diameter $\phi_{S-\text{nom}}$

The nominal hole diameter depends upon the desired clearance. i.e.:

$$\phi_{H-\text{nom}} = \phi_{S-\text{nom}} + \Delta_{desired} \qquad (1)$$

The maximum material sizes of the hole and the shaft depend upon the circular size tolerance:

$$\begin{aligned} \phi_{H-\min} &= \phi_{H-\text{nom}} - \varepsilon_{H-size} \\ \phi_{S-\max} &= \phi_{S-\text{nom}} + \varepsilon_{S-size} \end{aligned} \qquad (2)$$

We fabricate the shaft at some location (d_{S-nom}) with location tolerance ($\varepsilon_{S-location}$) for the position of the shaft axis with respect to a global datum frame.

The worst-case assemblability constraint is expressed in terms of the required location tolerance on the position of the hole axis at the *maximum-material condition* (MMC):

$$\varepsilon_{H-location} = \phi_{H-\min} - (\phi_{S-\max} + \varepsilon_{S-location}) \qquad (3)$$

Substituting from (Eq. 1) and (Eq. 2), it can be shown:

$$\varepsilon_{H-location} = \Delta_{desired} - (\varepsilon_{S-location} + \varepsilon_{S-size} + \varepsilon_{H-size}) \qquad (4)$$

If we assume that the achievable size tolerances on the shaft hole are the same, the above equation reduces to:

$$\boxed{\varepsilon_{H-location} = \Delta_{desired} - (\varepsilon_{S-location} + 2\varepsilon_{C-size})} \qquad (5)$$

We can also write the *least material condition* (LMC) sizes of the shaft and the hole as follows:

$$\begin{aligned} \phi_{H-\max} &= \phi_{H-\text{nom}} + \varepsilon_{H-size} \\ \phi_{S-\min} &= \phi_{S-\text{nom}} - \varepsilon_{S-size} \end{aligned} \qquad (6)$$

Noting that the maximum and minimum clearances occur when the features are at LMC and MMC respectively, we can derive expressions for the extrema of the clearance:

$$\begin{aligned} \Delta_{\max} &= \Delta_{desired} + \varepsilon_{H-size} + \varepsilon_{S-size} \\ \Delta_{\min} &= \Delta_{desired} - \varepsilon_{H-size} - \varepsilon_{S-size} \end{aligned} \tag{7}$$

Again, if we assume the same achievable size tolerance on both the hole and the shaft, we get:

$$\boxed{\begin{aligned} \Delta_{\max} &= \Delta_{desired} + 2\varepsilon_{C-size} \\ \Delta_{\min} &= \Delta_{desired} - 2\varepsilon_{C-size} \end{aligned}} \tag{8}$$

Method 2: Building the hole and shaft *in-situ*

In building a pin-joint using a layered fabrication technique, it is possible to reference the geometry of the second-built feature (hole or shaft) directly from a best- fit surface to the actual (or measured) geometry of the first-built feature. Since assemblability is no longer a constraint in the layered paradigm, looser location tolerance requirements (only to ensure that there is no material interference) will result. Additionally, if we assume the same process capability (in terms of the best possible hole and shaft size tolerances), better control over the clearance geometry is achieved using this method. These results are demonstrated below:

We choose the nominal shaft diameter $\phi_{S-\text{nom}}$

We build the shaft at some location ($d_{S-\text{nom}}$) with a location tolerance of $\varepsilon_{S-location}$. Alternately, we could choose to build the hole first.

We measure the resulting shaft using a touch probe and fit some approximating cylinder to it (we could use the Least Square or Chebychev fits depending upon the reliability of our measuring instrument [Feng and Hopp, 1991]). The resulting fit has a known axis location and diameter ($\phi_{S-\text{actual}}$). Note that the worst case shaft diameters are again given by:

$$\begin{aligned} \phi_{S-\max} &= \phi_{S-\text{nom}} + \varepsilon_{S-size} \\ \phi_{S-\min} &= \phi_{S-\text{nom}} - \varepsilon_{S-size} \end{aligned} \tag{9}$$

Next, we choose the nominal diameter of the hole based upon the clearance we need (similar to Method 1):

$$\phi_{H-\text{nom}} = \phi_{S-\text{actual}} + \Delta_{desired} \tag{10}$$

Instead of building the hole, we now build the gap out of sacrificial support material. This is done by depositing the sacrificial support in near-net shape around the shaft that has already been built, and machining the excess material. This is equivalent to building another shaft with diameter $\phi_{H-\text{nom}}$. Once this step is complete, the "hole" is deposited around the gap, and the sacrificial support is removed. The worst-case outer diameter of the support shaft is assumed equal to the worst-case inner diameter of the final hole and can be estimated as:

$$\begin{aligned} \phi_{H-\max} &= \phi_{H-\text{nom}} + \varepsilon_{S-size} \\ \phi_{H-\min} &= \phi_{H-\text{nom}} - \varepsilon_{S-size} \end{aligned} \tag{11}$$

We now note that the axis location of the sacrificial shaft just built could be off from the location of the original shaft axis by the <u>amount of the designed clearance</u> without any occurrence of interference (i.e. without undercutting into the actual shaft). Furthermore, the most conservative estimate of the location tolerance would be obtained when the sacrificial shaft is in its MMC. i.e.:

$$\varepsilon_{H-location} = \phi_{H-\min} - \phi_{S-\text{actual}} \tag{12}$$

Substituting from (Eq. 10) and (Eq. 11), it can be shown that:

$$\varepsilon_{H-location} = \Delta_{desired} - \varepsilon_{S-size} \tag{13}$$

An equivalent result would be attained if the hole were built first. If the achievable hole and shaft tolerances are assumed to be the same (ε_{C-size}), then the above equation can be re-written as:

$$\boxed{\varepsilon_{H-location} = \Delta_{desired} - \varepsilon_{C-size}} \tag{14}$$

We can derive the expressions for the maximum and minimum clearances achieved using this methodology as follows:

$$\begin{aligned} \Delta_{\max} &= \phi_{H-\max} - \phi_{S-\text{actual}} \\ \Delta_{\min} &= \phi_{H-\min} - \phi_{S-\text{actual}} \end{aligned} \tag{15}$$

Substituting again from (Eq. 10) and (Eq. 11), and assuming identical hole and shaft size tolerances:

$$\boxed{\begin{aligned} \Delta_{\max} &= \Delta_{desired} + \varepsilon_{C-size} \\ \Delta_{\min} &= \Delta_{desired} - \varepsilon_{C-size} \end{aligned}} \tag{16}$$

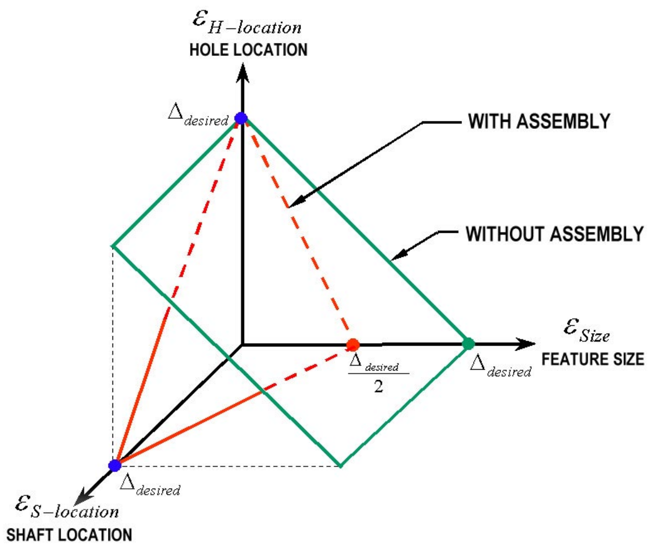


Fig. 10: Comparison of assembly and non-assembly methods – allowable tolerances to satisfy manufacturing and functional constraints

The above figure (Fig. 10) graphically illustrates the relationships derived in (Eq. 4) and (Eq. 14). Comparisons of the two methods can readily be made using (Eq. 4), (Eq. 8), (Eq. 14) and (Eq. 16). A smaller value for the location tolerance implies a tighter constraint (and superior process control), and is therefore undesirable. Note that the tolerance for the hole location using the second method is indifferent to the achievable tolerance on the shaft location.

Summarizing the previous analysis:

The required location tolerances for a shaft-hole pair when there are no assembly concerns present are always looser than the case where assemblability needs to be ensured (given the same desired clearance and process capability)

A more precise control is achieved on the magnitude of *clearance* between the hole and the shaft when there are no assembly concerns present. This is clearly preferable when the pin-joint is required to follow a designed trajectory within some error-limits.

CONCLUSIONS AND FUTURE WORK

It has been demonstrated that traditional tolerancing of mechanisms with mating and fitting surfaces is typically complicated by constraints that ensure interference-free assemblability. Layered manufacturing makes it possible to simultaneously shape and assemble parts *in-situ*, whereby the tolerances required to ensure desired behavior are looser, the location tolerances on complementary surfaces (e.g. the hole and shaft) are de-coupled from each other and better control is achieved over the behavior of the joint.

In building a tolerance representation that is to be used for the exchange of design information between designers and fabricators of layered parts, it is essential to exploit the advantages offered by simultaneous shape and assembly. In addition to permitting the specification of geometric tolerances as indicated in the ANSI and ISO standards, this representation should allow designers to specify limits on the variability of "shadow" geometry that typically does not appear explicitly in the geometric representation of the part. The example of a simple hinged mechanism is used to show how such a representation would offer designers better control over its behavior.

The results demonstrated in this paper clearly depend upon an assumption that the axis of rotation of the pin joint be parallel to the growth axis of the layered process. Given that a large number of mechanism assemblies being built using layered manufacturing today tend to be planar, 2 ½D structures, the result is significant. However, the issue of extending this analysis to the more general case of spatial mechanisms is important. In on-going work, we are developing a general specification for mating and fitting surfaces in layered manufacturing, suitable for designing mechanisms with standard kinematic pairs.

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