GPS Overview

Christopher R. Carlson
Dynamic Design Lab
Stanford University
03.01.02
Some Credit

• Most of this information is covered in AA272C with Professor Per Enge
  – A fun and not too difficult class for controls persons

• Many of these figures were borrowed from:
  – The Internet
  – Dr. Akos of the WAAS lab.
Outline

• Motivation
• Big picture
• RF signals
• Signal tracking
• GPS error sources
• Differential GPS
• Novatel Datasheet
Carlson’s Motivation

• GPS provides accurate (inertial) position and velocity information
• Some assumptions from classical estimation theory hold (it is a cool system)
Applications

• Vehicle state/parameter estimation
  – Velocity, attitude, etc
• (Precision) Navigation
  – Socially positive and negative
• Surveying unknown areas
  – Studying plate tectonics
• Studying the atmosphere
• Global time reference
GPS Satellite Constellation
Control Segment

Global Positioning System (GPS) Master Control and Monitor Station Network
GPS Position Solution

\[ \rho^{(k)} = \sqrt{(x^{(k)} - x)^2 + (y^{(k)} - y)^2 + (z^{(k)} - z)^2} - b \]

where \( k = 1, 2, ..., K \)

If \( K \geq 4 \), solve for user position \((x, y, z)\), and receiver clock bias \( b \)
Traveling at Light Speed

- GPS range measurements are the time of flight multiplied by the speed of light

- $3 \times 10^8 \ [\text{m/s}] \Rightarrow 1 \ \text{microsecond is} \ 300[\text{m}] \ \text{of error}$

- Need a very precise way to measure time
GPS time measurement

• The GPS signal extremely weak
• All signals ride the exact same carrier frequency
• Some clever EE’s applied CDMA techniques to GPS 30 years ago
CDMA uses PRN codes to “spread” a signal about the RF carrier

- Each “Chip” is 1 microsecond wide
- Sometimes Called the C/A code
PRN Autocorrelation

Cross Correlation of PRN 19 delayed 200
\[ \sin(a) \times \sin(b) = \sin(a+b) + \sin(a-b) \]
PSD of Carrier, PRN, Carrier x PRN

Magnitude dB

Frequency

-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
PSD of GPS Data
Measuring Time

• The entire GPS system is rising edge triggered with respect to GPS time standard
  – Satellites have very accurate clocks
• Accuracy of GPS time depends on how well receiver can resolve the rising edge of chip
• A rising edge is just a relative reference.
Navigation Message

- The PRNs are further encoded with a navigation message.
- Encodes the exact time of each chip transition
- Also contains satellite ephemeris (orbit) information
It gets a Little Complicated

- Need to remove carrier to resolve PRN
- Problem: Doppler
- Need know PRN to get carrier
- Upon startup, receiver goes through acquisition phase.
Coarse Acquisition (C/A)

- Searches over Frequency and PRN phase until close
- Code tracking loop
  - Local PRN synchronized with incoming PRN
- Carrier tracking loop
  - Reference carrier signal tracks incoming signal
Carrier and C/A search
Code Tracking, a DLL for the PRN

Autocorrelation of PRN30
A PLL for Carrier Tracking

Phase Detector

$F_{in}$  

LPF  

VCO  

$F_{out}$
Coupled Tracking Loops

- Carrier NCO
- PLL feedback control
  - carrier phase increment command
- Integrate & Dump Correlator
  - $I_E$
  - $I_P$
  - $I_L$
- Integrate & Dump Correlator
  - $Q_E$
  - $Q_P$
  - $Q_L$
- Code Generator
- Code NCO
- EPLL shift register
- DLL feedback control
  - code phase increment command

Microprocessor
- Search
- Acquisition
- PLL discriminator control cmd
- DLL discriminator control cmd
- Lock Detector
- SNR estimation
- Navigation

Digital Samples @ IF from front-end ASIC
Loop Bandwidths

• Carrier tracking loop usually 10-100hz
  – Trade off tracking with noise rejection
• Novatel claims a BW of 15hz. But gives solutions at 20hz.

• Code tracking loop (DLL) usually 0.1-10hz
  – Code loop aided by carrier PLL
What Was All That?

• Receiver clicks on
• Searches for satellites, locks on
• Decodes navigation message and figures out what time signal was sent
• Zooms in on a chip transition and very precisely time stamps it
• Difference between satellite and receiver times multiplied by “c” gives pseudorange
Dominant Error Sources

- Iono & Tropo Act Like Lenses
Selective Availability

- DOD deliberately dithered the satellite clocks
- Position Error an order of magnitude higher
- Dither was pretty much white
- Civilian DGPS pretty much kicked it
## GPS Error Sources

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Typical Range Error</th>
<th>DGPS (Code) Range Error &lt;100 km Ref-Remote</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV Clock</td>
<td>1 M</td>
<td>1 M</td>
</tr>
<tr>
<td>SV Ephemeris</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>Selective Availability</td>
<td>10 M</td>
<td></td>
</tr>
<tr>
<td>Troposphere</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>Ionosphere</td>
<td>10 M</td>
<td></td>
</tr>
<tr>
<td>Pseudo-Range Noise</td>
<td>1 M</td>
<td>1 M</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>1 M</td>
<td>1 M</td>
</tr>
<tr>
<td>Multipath</td>
<td>0.5 M</td>
<td>0.5 M</td>
</tr>
<tr>
<td>RMS Error</td>
<td>15 M</td>
<td>1.6 M</td>
</tr>
<tr>
<td>Error * PDOP=4</td>
<td>60 M</td>
<td>6 M</td>
</tr>
</tbody>
</table>

PDOP = Position Dilution of Precision (3-D) 4.0 is typical
Differential GPS
Differential GPS

- Often uses a base station with a surveyed position.
- Allows for common mode errors to be canceled right away.
- Requires some sort of information transfer
  - Radio
  - Post processing
  - WAAS
Differential GPS

• Carrier Phase GPS uses the PLL cycle count and the wave length of the carrier to infer distance.
• New problems
  – Integer ambiguity
  – Cycle slips
• Real time centimeter level accuracy
Why is Velocity So Good?

• Velocity uses pseudorange rates
  – Automatically ignores slowly varying errors such as Ionosphere and Troposphere
• Can get rate information from the Doppler in the PLL.
• Each carrier cycle ➔ 19 cm.
• Similar equation form to position solution
### OEM4 Specifications

<table>
<thead>
<tr>
<th>Performance</th>
<th>Time To First Fix</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Position Accuracy</strong></td>
<td><strong>Cold Start</strong></td>
</tr>
<tr>
<td>Single Point L1</td>
<td>1.8 m CEP</td>
</tr>
<tr>
<td>Single Point L1L2</td>
<td>1.5 m CEP</td>
</tr>
<tr>
<td>WAAS L1</td>
<td>1.2 m CEP</td>
</tr>
<tr>
<td>WAAS L1L2</td>
<td>0.8 m CEP</td>
</tr>
<tr>
<td>DGPS (L1, C/A)</td>
<td>0.45 m CEP</td>
</tr>
<tr>
<td>RT20&lt;sup&gt;2,3&lt;/sup&gt;</td>
<td>&lt; 20 cm CEP</td>
</tr>
<tr>
<td>RT2&lt;sup&gt;3&lt;/sup&gt;</td>
<td>1 cm + 1 ppm</td>
</tr>
<tr>
<td><strong>Measurement Precision</strong></td>
<td><strong>Signal Reacquisition</strong></td>
</tr>
<tr>
<td>L1 C/A Code</td>
<td>6 cm RMS</td>
</tr>
<tr>
<td>L2 P Code</td>
<td>25 cm RMS (AS on)</td>
</tr>
<tr>
<td>L1 Carrier Phase</td>
<td>0.75 mm RMS</td>
</tr>
<tr>
<td>(differential channel)</td>
<td></td>
</tr>
<tr>
<td>L2 Carrier Phase</td>
<td>2 mm RMS</td>
</tr>
<tr>
<td>(differential channel)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Typical values. Performance specifications are subject to GPS system characteristics, U.S. DOD operational degradation, ionospheric and tropospheric conditions, satellite geometry, baseline length and multipath effects. Assumes SA Off.
2. Expected accuracy after three minute static convergence over short baseline.
3. See Typical Performance charts below. Subject to convergence time and baseline length.