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USING MEMS ACCELEROMETERS TO IMPROVE AUTOMOBILE HANDWHEEL STATE ESTIMATION FOR FORCE FEEDBACK

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ABSTRACT

This paper explores the possibility of combining solid state accelerometers with a low resolution position sensor to provide clean estimates of automobile handwheel position, velocity and acceleration for use in force feedback. Typically determining the acceleration and velocity of the handwheel requires differentiating a position sensor such as an encoder or potentiometer. Unless expensive high-resolution sensors are used, this differentiation leads to a noisy signal, requiring significant filtering which leads to significant phase lag. With a direct measurement of acceleration, we circumvent many of the problems associated with differentiation and filtering. This work uses a Kalman filter to combine a pair of MEMS accelerometers with a low-resolution potentiometer to estimate handwheel states. This measurement scheme is effective in this application because of the low frequency nature of the force feedback and because of the structural robustness of the handwheel system. Initial in-vehicle experimental results show the setup can provide smooth acceleration and velocity signals in a moving vehicle.

NOMENCLATURE

δ Handwheel angle
b Total accelerometer bias
A_d Discrete state transition matrix
B_d Discrete input matrix
C_d Discrete output matrix
L_{ss} Steady state Kalman gain
Σ_{ss} Steady state error covariance
Σ_v Sensor noise covariance
Σ_w Process noise covariance
x Estimate state vector

INTRODUCTION

The handwheel is the primary control mechanism of an automobile, and thus interaction between the handwheel and the driver is critical to safety. Forces on the handwheel communicate to the driver about the tire forces acting on the vehicle. The handwheel also presents a predictable mechanical feel to the driver to allow smooth and safe control.

In a vehicle with a conventional steering system, forces are transmitted from the road wheels to the handwheel through the mechanical steering linkage. These include forces from road imperfections and the aligning moment. The aligning moment is what causes the steering wheel to return to center while the vehicle is moving. It is caused by the offset between the steering axis and the application point of the tire side force in the contact patch (see Figure 1). This offset is a combination of the mechanical and pneumatic trails. The pneumatic trail is the offset between the center of the tire patch and the centroid of the side force generated by the tire. This is a result of the mechanism by which tires generate lateral force. The mechanical trail is a geometric offset caused by the design of the steering geometry. The effective lever arm is the sum of these two offsets, and the tire

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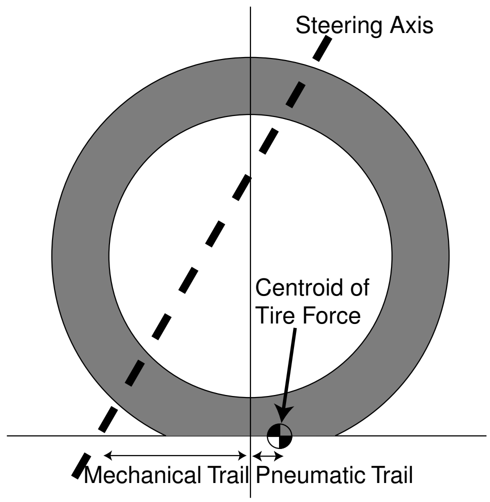

Figure 1. STEERING GEOMETRY CAUSING ALIGNING MOMENT.

force applied about this arm causes a moment about the steering axis that tends to straighten the wheels. In a conventionally steered vehicle, in addition to acting to straighten the wheels, this moment also provides the driver an indication of the forces on the road wheels, and can warn of an impending loss of traction.

Steer-by-wire vehicles require artificial force feedback to replicate these forces in a conventional steering system. Steer-by-wire differs from conventional steering in the way the handwheel controls the roadwheels. In a conventionally steered vehicle, the handwheel is mechanically connected to the road wheels, and the driver controls the steering system of the vehicle through this mechanical linkage. In a steer-by-wire vehicle this mechanical link has been removed, and some sort of actuator controls the road wheels in response to commands from the handwheel (see Figure 2). Steer-by-wire systems of the future will increase vehicle safety, simplify vehicle cabin design, and open up new possibilities for vehicle control.

With the mechanical steering link removed, any force feedback at the handwheel must be artificially created using a motor or other actuator. This includes the aligning moment, but must also include an appropriate amount of damping and inertia to

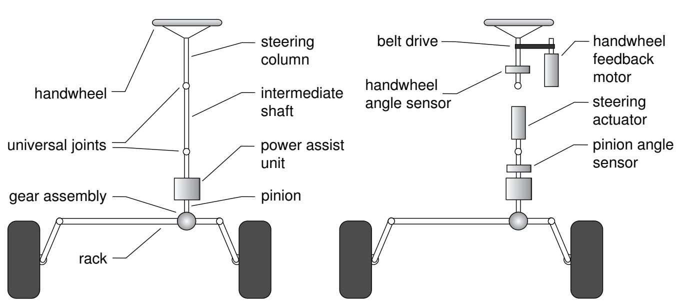

Figure 2. STEER BY WIRE CONCEPT.

allow smooth control of the vehicle. One way to recreate this damping and inertia is to estimate the velocity and acceleration of the handwheel and provide force feedback to counteract this motion. Similarly, the aligning moment can be recreated with accurate knowledge of handwheel position and vehicle motion.

Thus to create this force feedback we need accurate estimates of three handwheel states: position, velocity and acceleration. These states are also needed for accurate feedforward control for steer-by-wire, in which the handwheel command and its derivatives are used to compensate for the dynamics of the steer-by-wire system [1]. Traditional automotive handwheel angle sensors are low resolution encoders or potentiometers, and are inadequate for this task as they contain too much noise or discretization error to allow derivatives. Although sufficient for position sensing for applications like vehicle stability control, this high frequency noise makes these measurements useless for high bandwidth estimation of velocity or acceleration.

Most haptic systems (in laboratories or some consumer products) use high resolution encoders to measure shaft positions. The benchmark laboratory haptic interface, the PHANTOM, for example, uses several brushed DC motors, each with a high resolution encoder [2]. These sensors are excellent for this task because when used properly they have no electrical noise, so the only measurement error comes from the quantizing of position. With each derivative of this position signal the quantization error is increased. However, if of a high enough resolution, this quantization is a minor effect and two derivatives of position can be taken without inducing overly large amounts of noise. Unfortunately, high resolution encoders are expensive because of the precision of the mechanical and optical components needed. In addition, because they measure position instead of velocity or acceleration, a sensor with small enough quantization error to provide reasonable velocity and acceleration estimates will have a positioning precision way beyond what is necessary.

With the rapid advance of MEMS devices in recent years, extremely cheap solid-state accelerometers are now available. Because these are compact, inexpensive, and provide a direct measurement of acceleration, they are attractive for automotive use. These accelerometers can directly measure the acceleration on the handwheel, allowing high bandwidth estimation of both acceleration and velocity of the handwheel.

Most haptic systems do not use an accelerometer to estimate acceleration or velocity because of the high frequency noise introduced by directly measuring acceleration. Instead, the usual practice is to combine a position sensor with a model of the dynamics of the system to help estimate the states and/or external forces being applied to the system [3]. There are two key differences between the handwheel application and traditional haptic setups which might allow accelerometers to be useful where they are normally not even considered. First, the dynamics of the handwheel and aligning moment are both at low frequencies, below about 5Hz. Thus the force feedback need not have high frequency components much above this range. Haptic systems are also designed to be transparent to the user, presenting only the forces desired to represent the slave system with minimal inertia on the master. This is done by making the master as light as possible, quite unlike the robust steering wheel in most cars. A steering wheel has significant inertia, which acts as a mechanical filter to remove high frequency vibration. This inertia is acceptable because this work seeks to recreate the feel of a handwheel, which includes a feeling of inertia. For these two reasons accelerometers may work in a vehicle application.

Accelerometers have been successfully used before in high bandwidth control. Fassnacht and Mutschler [4] used an accelerometer for high speed motor control when a differentiated encoder would have too much noise. To date, no one has applied accelerometers for handwheel feedback in steer-by-wire. Instead, most work on force feedback for steer-by-wire focuses on complex modeling to reproduce the nonlinear characteristics felt in a conventional vehicle [5], [6], [7], [8]. This modeling can be combined with control of the steer-by-wire itself as in work by Setlur et al. [9], in which a nonlinear tracking controller is used to simultaneously control the steer-by-wire and force feedback. Here we are concerned not with complicated forms of force feedback, but with state estimation to allow low frequency force feedback.

This work explores whether accelerometers are useful for handwheel state estimation. A Kalman filter combines the accelerometers with an absolute position potentiometer. The state estimates resulting from this filter can be used to provide force feedback to recreate inertia and damping. Experimental in-vehicle results show that the approach works well at low frequencies, but that high frequency noise and instability can be a problem when trying to recreate the feel of a high level of inertia. At the level of inertia and damping needed to provide a comfortable feel to the driver, this noise is not a problem.

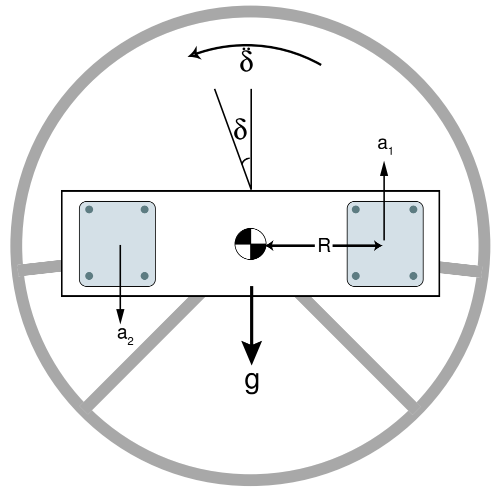

Figure 3. MEASUREMENT SETUP ON HANDWHEEL.


Figure 4. HANDWHEEL POSITION IN VEHICLE.

MEASUREMENT CONFIGURATION
Physical Setup

To estimate the handwheel position, velocity and acceleration, we want to measure angular acceleration and combine this with a low resolution position measurement using a Kalman filter. To produce a useful angular acceleration signal, the accelerometers are mounted as shown in Figure 3, in which the arrows indicate the direction of measurements a_1 and a_2. The sensors measure zero acceleration when oriented horizontally. By orienting the sensors so that the measurement directions oppose each other and summing their signals, the specific force due to gravity can be cancelled.

$$a_1 = R\ddot{\delta} + g cos(\delta) \quad (1)$$
$$a_2 = R\ddot{\delta} - g cos(\delta) \quad (2)$$

Combining equations 1 and 2 and dividing by 2 yields

$$\ddot{\delta} = \frac{a_1 + a_2}{2R} \quad (3)$$

This signal summation should cancel any acceleration common to the two sensors, such as gravity or vehicle body acceleration.

Error Sources

Ideally, the setup described above should cancel gravity completely and would measure pure rotational acceleration. In reality a number of error sources may affect the measurements. The cancellation of gravity is achieved only if several conditions are met. First, the sensors must both be measuring in the plane of the handwheel rotation. In this way as the handwheel is turned the two sensors will always measure equal but opposite components of gravity, allowing cancellation. If the sensitivity is slightly different between the two sensors, a similar error will result in the form of an uncancelled gravity component dependent on handwheel position.

In addition to inaccuracies resulting from sensor placement, it is possible that vehicle dynamics will couple with the system and prevent an accurate determination of the handwheel states. Of concern are accelerations in roll, yaw, and pitch. Since most steering wheels are close to vertically aligned (see Figure 4), yaw and pitch will not significantly affect the measurement, because the motion is nearly perpendicular to the direction of measurement. In addition, typical yaw and pitch accelerations are about $30deg/s^2$, much smaller than the handwheel accelerations of interest. Vehicle roll is of greater concern as it is roughly in the same plane as the handwheel acceleration. The angle of the steering wheel will not minimize the effects of roll significantly as in the case of yaw and pitch. Thus any vehicle roll acceleration will appear as handwheel acceleration to the inertial sensors. However, amounts of roll even in aggressive slalom maneuvers are on the order of $30deg/s^2$, again quite small in comparison to those found for the handwheel acceleration during aggressive maneuvers.

Structural vibrations of the handwheel system are also a concern. Realistically, any fixture in a car will have some high frequency modes that will appear when measuring acceleration using high bandwidth accelerometers. The relative phase of these structural vibrations between the two sensors depends on the mode of vibration and thus may not be cancelled by summing the signals. Fortunately, if the structure is rigid enough the resonances will be high enough frequency that they can be filtered without adding significant phase lag to the handwheel acceleration.

SIGNAL PROCESSING: THE KALMAN FILTER

To combine the potentiometer with the accelerometers, a Kalman filter is used. The Kalman filter was chosen over a traditional observer so that knowledge of the noise magnitudes could be used to find optimal filter gains. The Kalman filter simultaneously combines the two measurement sources and estimates any bias in the acceleration measurements so this bias can be removed.

The only model required for the Kalman filter is the kinematic relationship between the handwheel acceleration and the handwheel position.

$$\begin{bmatrix} \dot{\delta} \\ \ddot{\delta} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \dot{\delta} \\ b \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \ddot{\delta}_{meas} \tag{4}$$

Here δ is the angle of the handwheel, and b is the combined bias of the two accelerometers.

The Kalman filter consists of a time update step and a measurement update step. In the time update step, the dynamics of the system are propagated forward. This is followed by the measurement update step where the propagated state is compared to a new measurement and corrections are made. In this case, the acceleration is used to propagate the state forward and the potentiometer is used for the measurement update. The Kalman filter used assumes steady state values of the covariances, and thus the measurement and time updates can be collapsed into a single equation:

$$x_+ = A_d x_- + L_{ss}(\delta_{pot} - C_d x_- + B_d \ddot{\delta}_{accel}) \tag{5}$$

Here A_d is a forward euler discretized version of the A matrix in equation 4 (with sampling time T_s), and x_+ and x_- are the current and previous state estimate vectors.The C_d matrix is simply a row vector stating that the potentiometer is measuring δ:

$$A_d = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix} \qquad B_d = \begin{bmatrix} 0 \\ T_s \\ 0 \end{bmatrix} \qquad C_d = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{6}$$

The Kalman filter gain L_{ss} is found from the steady state covariances of the states as

$$L_{ss} = A_d \Sigma_{ss} C_d^T (C_d \Sigma_{ss} C_d^T + \Sigma_v)^{-1} \tag{7}$$

Σ_{ss} is the steady state covariance of the state estimate, which is found as the solution to the discrete algebraic Riccati equation

$$\Sigma_{ss} = A_d \Sigma_{ss} A_d^T + \Sigma_w - A_d \Sigma_{ss} C_d^T (C_d \Sigma_{ss} C_d^T + \Sigma_v)^{-1} C_d \Sigma_{ss} A_d^T \tag{8}$$

Here Σ_w is the process noise covariance, and Σ_v (a scalar) is the measurement noise of the potentiometer. All process noise covariance values are set to zero except the accelerometer noise and bias variance.

$$\Sigma_w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{acc} & 0 \\ 0 & 0 & \sigma_{bias} \end{bmatrix} \tag{9}$$

These variance values are shown in Table 1. The accelerometer and potentiometer noises are simply the variance of data measured with the handwheel stationary and the engine running but with the vehicle not moving. The bias variance is a tuning parameter for the filter. The actual bias of the sensors does not change nearly as quickly as the variance of Table 1 would indicate. In actual filter operation, this bias term compensates for un-modeled dynamics in the system in addition to electrical biases.

Table 1. KALMAN FILTER NOISE PARAMETERS.

Variance	Symbol	Value	Units
Accelerometer Noise	σ_{acc}	200	deg^2/s^4
Bias Variance	σ_{bias}	100	deg^2/s^4
Potentiometer Noise	Σ_v	0.048	deg^2

IMPLEMENTING FORCE FEEDBACK

To evaluate the effectiveness of this measurement setup, inertia and damping are recreated on the handwheel of the vehicle. As shown in Figure 4, a brushless DC motor is connected to the handwheel via a 5:1 ratio synchronous belt drive to provide force feedback torque. This motor can supply a peak of 20Nm while the handwheel is spinning at 700deg/s. It is also equipped with a high resolution shaft encoder, with a resolution of 1000 ticks per revolution at the motor, or about 5000 ticks per revolution at the handwheel. The high precision of this device allows it to be used as a truth reference for the measurement scheme presented in this paper.

The accelerometers used are MEMS units from Analog Devices with a measurement range of 1.7g and a user selectable output bandwidth, set to 200Hz. The mounting in the vehicle is shown in Figure 4. It consists of a rigid mounting plate with the accelerometers about 5cm from the axis of rotation. With the radius of 5cm, these sensors can measure a maximum angular acceleration of about $8000 deg/s^2$. This is about a factor of 1.5 above the maximum acceleration seen in even emergency maneuvers.

To recreate the feeling of viscous damping, a torque command is sent to the handwheel in opposition to the velocity.

$$\tau_{damping} = -C_{damp}\dot{\delta}_{est} \tag{10}$$

Where $\dot{\delta}_{est}$ is the velocity estimate output of the Kalman filter. C_{damp} is a constant representing the amount of viscous damping recreated by the force feedback.

To recreate inertia, the acceleration signal from the sum of the two accelerometers is used to counteract measured acceleration:

$$\tau_{inertia} = -I_{add}\ddot{\delta}_{est} \tag{11}$$

Here I_{add} is the amount of inertia to be added by the force feedback system. $\ddot{\delta}_{est}$ is the acceleration signal from the sensors with the estimated bias removed. In this way, the small offset in acceleration will not cause a constant offset in torque from the artificial inertia. These estimates are fed back directly. There is no model of handwheel position between the state estimates and the torque command to the motor, so lag is minimized.

STATIONARY TESTING

Before evaluating this measurement setup in a moving vehicle, it is useful to evaluate its performance in a stationary vehicle. This allows separation of the effects of vehicle motion and vibration from inherent limits of the measurement scheme.

The amount of inertia recreated is chosen to feel like a stock Corvette. Through a system identification of the steering system, the inertia as seen at the input shaft of the power steering is $0.0285 kgm^2$. The inertia added is chosen to be $0.009 kgm^2$ to make the total when combined with the motor system match that of the stock Corvette. The damping is chosen to feel reasonable to the user, at a value of $0.172 Nm/rad/s$. The inertia signal is filtered at 20Hz to remove any high frequency signal which might cause unwanted vibration.

Figure 6 shows the acceleration, velocity, position and bias estimate with the user turning the handwheel to simulate driving. Not shown is the first few seconds of data in which the bias quickly converges to the actual bias, successfully removing any offset from the acceleration signal. This convergence takes about $0.2s$.

The filter effectively smooths the position signal from the potentiometer, with no perceptible lag induced. In fact at the scale shown the filtered and raw signals are indistinguishable. In the velocity signal we see the benefit of the estimation scheme. The velocity estimate is very smooth, and has no significant lag when compared to a differentiated version of the potentiometer signal. The potentiometer velocity presented is more noisy despite being filtered at 40Hz, and actually lags behind the Kalman filter output. Thus the velocity estimate is as good or better than that from a typical potentiometer. The acceleration signal contains significant high frequency content, but is actually much cleaner than a double differentiated and filtered encoder signal. The bias does drift with handwheel position, but the magnitude of this drift is much smaller than the signal itself. Because the vehicle is not moving for this test, this drift is not from vehicle motion but rather is due to inaccuracy in the mounting or sensitivity of the sensors. Because the filter successfully removes this drift as a bias, it does not noticeably affect the velocity or position estimates. This confirms that the inevitable small mounting errors associated with a real-world implementation can be tolerated by this measurement scheme.

EXPERIMENTAL RESULTS IN MOVING VEHICLE

Installation of this measurement setup in a steer-by-wire vehicle allows for evaluation of its performance on the road and with a human in the loop. The test vehicle is a 1997 Corvette modified to a full steer-by-wire setup (no mechanical connection between the road wheels and the handwheel) [1]. A brushless DC motor drives the input shaft of the power steering unit to steer the vehicle in response to commands from an onboard computer.

Testing while driving

The next question experimentally is whether the estimation scheme still works well when the vehicle is in motion. As detailed earlier, vehicle motion and vibration will enter the measurements. Here we seek to examine the magnitude of this noise, and whether it is amplified by the estimation scheme. Figure 7 shows the estimates while the driver is controlling the vehicle through maneuvers typical of city driving. These tests were performed at speeds of 12-15m/s on the West ramp and parallel of Moffett Federal Airfield. This surface includes potholes and pavement seams as large as those on bumpy city streets. The suspension of the Corvette is rather stiff, transmitting large shock and vibration to the entire vehicle. Despite this, the estimates are almost as clean as those of Figure 6 in which the vehicle was not moving. The velocity and position estimates effectively remove most of the vibrations, resulting in estimates similar to those in the stationary vehicle. The acceleration signal, however, does show more high frequency content in the 30-50Hz range with the vehicle in motion. Because this signal appears in both the accelerometer signal and the twice-differentiated encoder signal, it represents actual motion, probably due to engine vibration transmitted to the handwheel. The vibration signal is small compared to the main acceleration signal and is not a problem for low frequency force feedback. Vehicle vibrations therefore are the effective limit on the bandwidth of force feedback that can be applied to the handwheel.

Autonomous Maneuvers

The above experiments show that the system works well when the user is actively steering. The system must also work with hands off the handwheel. The concern here is that the handwheel would move due to noise or other excitation, causing vehicle instability. To evaluate this scenario, the car's steer-by-wire system can steer in the absence of handwheel command with the handwheel force-feedback active. In this way, both of the major noise sources will be present: vehicle motion and vehicle vibration.

Figure 5. STEER-BY-WIRE CORVETTE.

Figure 8 shows two types of autonomous driving. Column (a) shows the estimates when the vehicle is driven straight, but over some large bumps and potholes. Column (b) shows the estimates when the roadwheel angle is a sinusoid of magnitude 20degrees and frequency of 2rad/s. These two tests allow the separation of effect from bumps and vehicle motion. The state estimates are largely uncorrupted by either the bumps or vehicle motion. With the sinusoidal steering, the bias does drift somewhat from the vehicle motion, but this drifting is about two orders of magnitude smaller than the acceleration signal itself. This is approximately the magnitude of the vehicle roll acceleration, which is indistinguishable from handwheel acceleration with this measurement scheme as discussed in the section on error sources. The high frequency vibration noticed with the user steering is present in the acceleration signal, but it is not of a large enough magnitude to move the handwheel. This shows that the system is stable with the user's hands off the handwheel.

CONCLUSIONS AND FUTURE WORK

These experiments show that accelerometers can be used to effectively estimate handwheel states in a moving vehicle. The measurement setup cancels gravity and minimizes the effect of vehicle motion on the state estimates. Vehicle roll does appear in the accelerometer bias estimate but does not cause noticeable torque on the handwheel.

Force feedback will be a critical component of future vehicle control applications of steer-by-wire. The state estimates obtained here can be used for a variety of force feedback schemes. Advanced stability control, lanekeeping assistance and other control schemes made possible by steer-by-wire all require successful interaction with the driver, which requires careful design of the force feedback. With effective handwheel force feedback and advanced steer-by-wire algorithms, the cars of the future will be higher performance, safer, and more fun to drive.

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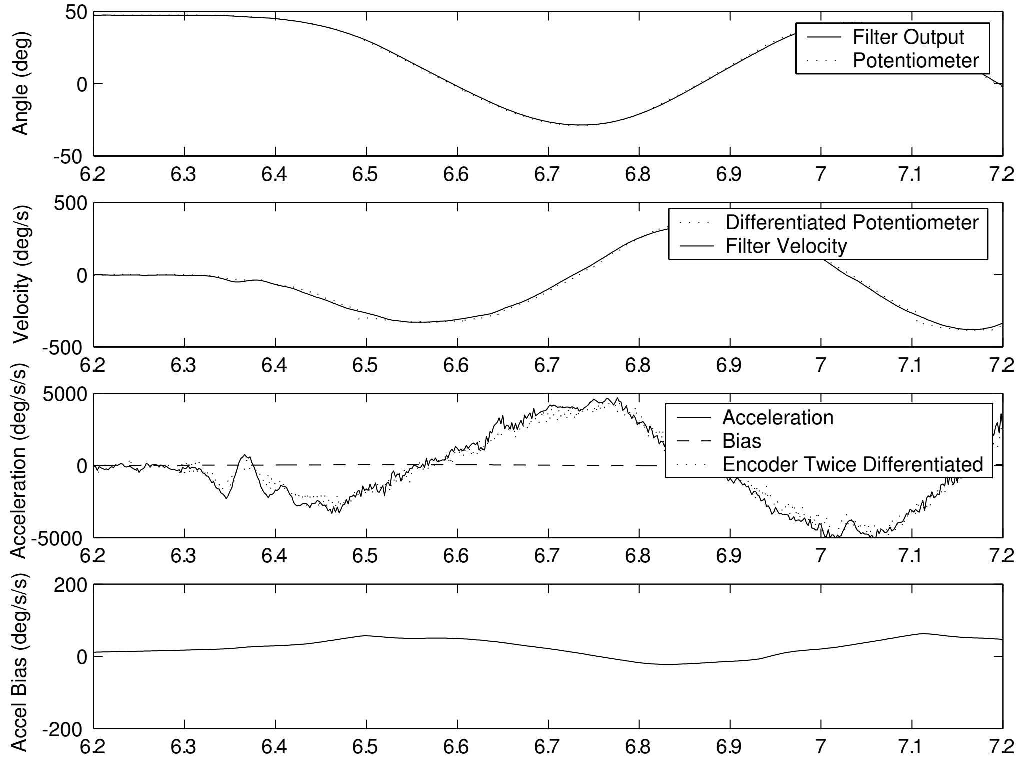

Figure 6. STEERING IN STATIONARY RUNNING VEHICLE.

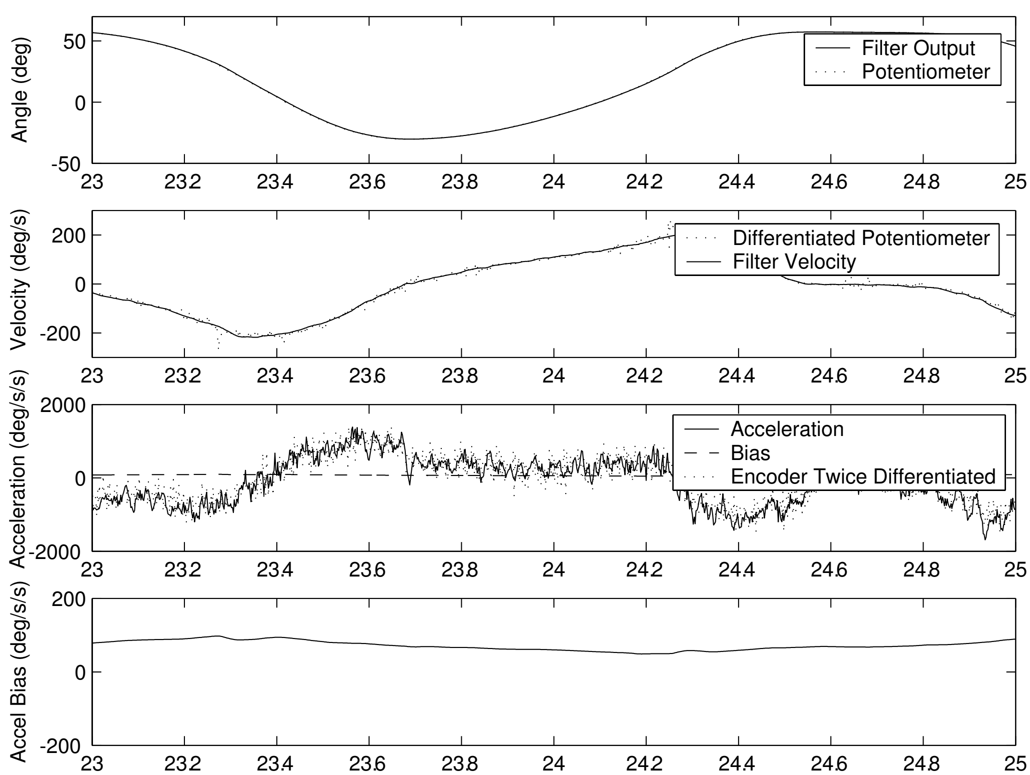

Figure 7. VEHICLE MOVING WITH TYPICAL USER STEERING.

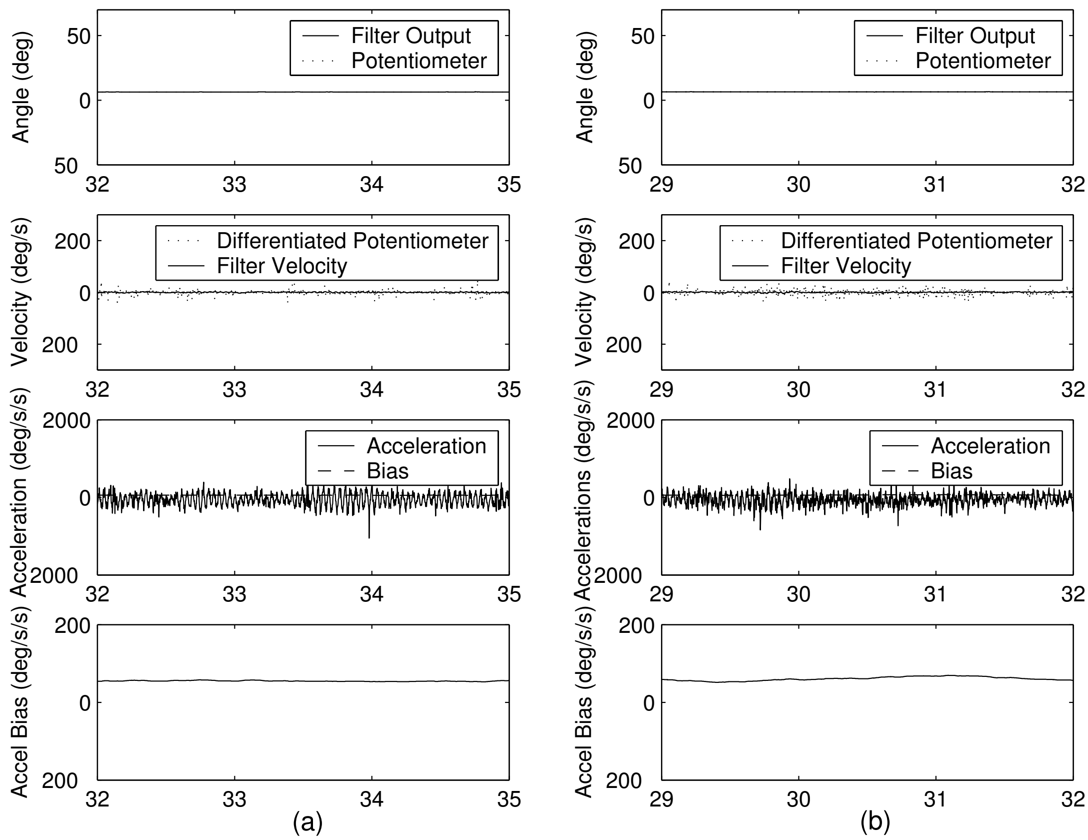

Figure 8. VEHICLE MOVING AND STEERING WITH NO HANDWHEEL MOTION.

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REFERENCES

[1] Yih, P., Ryu, J., and Gerdes, J., 2003. "Modification of vehicle handling characteristics via steer-by-wire". In Proceedings of the American Control Conference, pp. 2578–2583.

[2] Massie, T. H., and Salisbury, J. K., 1994. "The phantom haptic interface: A device for probing virtual objects". In Proceedings of the ASME Winter Annual Meeting, Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems.

[3] Sirouspour, M. R., DiMaio, S. P., Salcudean, S. E., Abolmaesumi, P., and Jones, C. "Haptic interface control– design issues and experiments with a planar device". In Proceedings of the IEEE International Conference on Robotics and Automation, pp. 789–794.

[4] Fassnacht, J., and Mutschler, P., 2001. "Benefits and limits of using an acceleration sensor in actively damping high frequent mechanical oscillations". In Proceedings of IEEE-IAS 2001 Chicago USA, pp. p.2337–2344.

[5] Tokunaga, H., Misaji, K., Takimoto, S., Shimizu, Y., and Shibata, K., 2002. "Steer feel evaluation method based on 'analytical method of equivalent linear system using the restoring force model of power function type"'. In Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), pp. 849–854.

[6] Segawa, M., Kimura, S., Kada, T., and Nakano, S., 2002. "A study of reactive torque control for steer by wire system". In Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), pp. 653–658.

[7] Ueda, E., Inoue, E., Saki, Y., Hasegawa, M., Takai, H., and Kimoto, S., 2002. "The development of detailed steering model for on-center handling simulation". In Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), pp. 657–662.

[8] Sakai, H., and Higuchi, A., 2000. "A theoretical vehicle model for design and evaluation of on-center handling". In Proceedings of the International Symposium on Advanced Vehicle Control (AVEC).

[9] Setlur, P., Dawson, D., Chen, J., and Wagner, J., 2002. "A nonlinear tracking controller for a haptic interface steer-by-wire systems". In Proceedings of the American Control Conference.