The steering model

In a conventional automotive steering system, the steering wheel is mechanically linked to the front wheels. Steer-by-wire removes this mechanical connection and instead, electronically controls the steering angle based on measurement of the handwheel angle. A model of the steer-by-wire implementation is shown below.

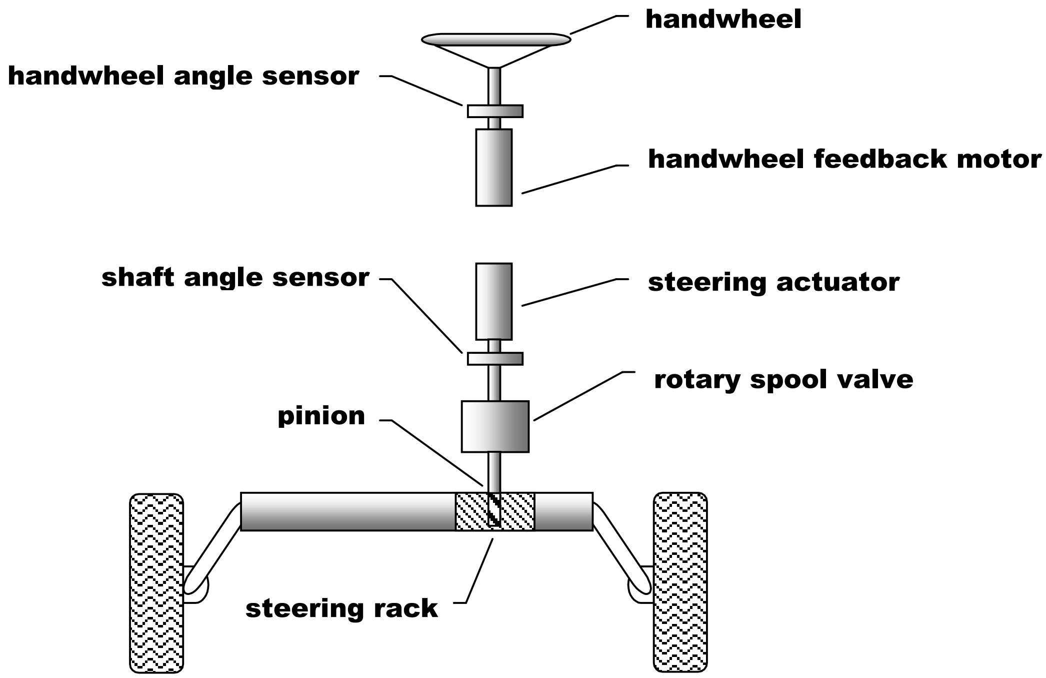

Figure 1: Steer-by-wire schematic.

The steering actuation system consists of the actuator, shaft angle sensor, and rotary spool valve (an integral part of the hydraulic power assist unit). Together with the rack and pinion, steering linkages, and front wheels, the combined system can be approximated by a two degree of freedom model. The actuator rotor inertia and upper shaft inertia are lumped into a single inertia. The pinion inertia, rack mass, steering linkage inertias, and wheel inertias are lumped together into a separate inertia. For the sake of convenience, the inertia is defined about the axis of steering rotation of the front wheels. The torsion bar, which is inside the rotary spool valve, connects the two masses and allows them to move relative to each other. Figure 2 shows a physical description of the steering actuation system as implemented in the test vehicle. Note that the motor output shaft, coupling, and universal joint are rigidly connected in this model and considered a part of the upper shaft inertia.

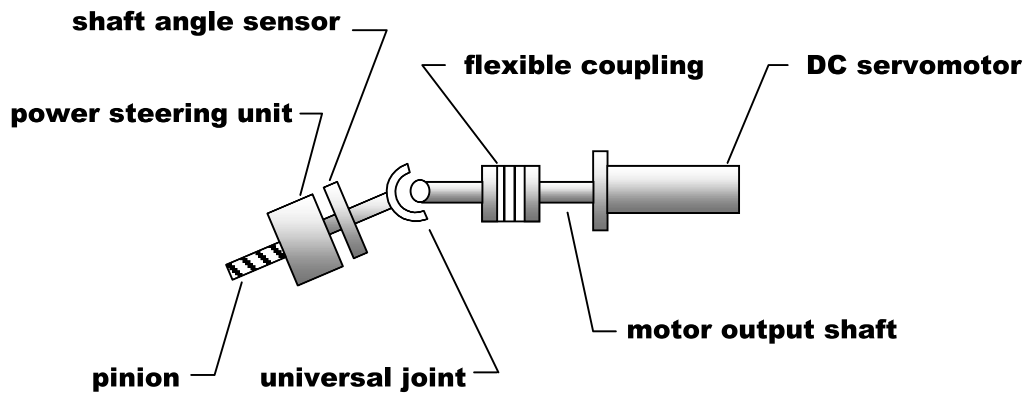

Figure 2: Steering actuation system side view.

The handwheel feedback system consists of the handwheel, handwheel angle sensor, and torque feedback motor. The purpose of handwheel torque feedback is to communicate to the driver via tactile means the direction and level of forces acting between the front tires and the road. A byproduct of these forces is the self-centering effect that occurs when the driver releases the steering wheel while exiting a turn. Both the self-centering effect and the torque feedback are important characteristics that a driver expects to feel when steering a car equipped with a conventional steering system.

Steering actuator

The actuator is a brushless DC servomotor described by the simplified DC motor equations.

$$I_m = k_I \tau_m$$
$$V_m = I_m R + k_E \omega_m$$

The motor torque and speed are related to torque and speed at the output shaft by the gear reduction ratio and gearhead efficiency.

$$\tau_m = \frac{1}{\eta}\frac{\tau_s}{r_g}$$
$$\omega_m = \dot{\theta}_s r_g$$

The motor and gearhead combination installed in the test vehicle was selected based on the maximum torque and speed necessary to steer the vehicle under typical driving conditions including moderate emergency maneuvers. Studies have suggested that required steering torque at the handwheel during normal driving ranges from 0 to 2 Nm, while emergency maneuvers can demand up to 15 Nm of torque [1]. The steering rate target was one full turn of the steering wheel per second. From these target values, the maximum current and voltage necessary to run the motor were calculated using the motor and gear equations.

symbol	description
θ_s	output shaft angle (rad)
τ_s	output shaft torque (N m)
ω_m	motor speed (rad/s)
τ_m	motor torque (N m)
I_m	motor current (A)
V_m	motor voltage (V)

Table 1: Motor and gearhead variables.

symbol	description	value
k_I	current constant (A/(N m))	68.7
k_E	back-EMF constant (V/(rad/s))	0.0145
R	terminal resistance (Ω)	0.6
r_g	gear reduction ratio	246
η	gearhead efficiency	0.6

Table 2: Motor and gearhead parameters.

Universal joint

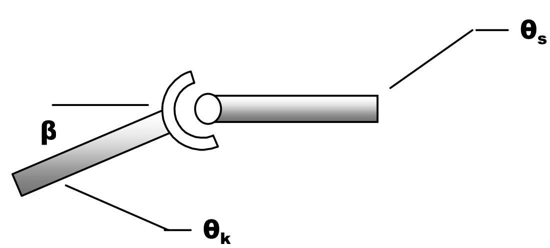

Figure 3: Universal joint.

Due to space constraints in the engine compartment of the test vehicle, the motor cannot be mounted along the same axis as the power steering input shaft. The addition of a universal joint, while permitting transmission of torque between two non-collinear axes, introduces slight variations in the transmitted torque and shaft rotation that are functions of the angle β between the two axes.

$$\theta_k = \tan^{-1}\left(\frac{\tan\theta_s}{\cos\beta}\right)$$

$$\dot{\theta}_k = \left(\frac{\cos\beta}{1-\sin^2\beta\cos^2\theta_s}\right)\dot{\theta}_s$$

$$\tau_s = \left(\frac{\cos\beta}{1-\sin^2\beta\cos^2\theta_s}\right)\tau_k$$

symbol	description	value
β	universal joint angle (deg)	20

Table 3: Universal joint angle.

A flexible coupling—rigid in torsion but compliant in bending—accommodates any axial misalignment between the connecting shafts.

Hydraulic power assist

The original hydraulic power assist unit in the test vehicle was retained as part of the steer-by-wire system. The incorporation of the stock power assist unit eliminates the need for extensive modifications to the existing steering system and allows the use of a much smaller actuator since the assist unit provides a majority of the steering effort. The only drawback is that the hydraulic system has its own dynamics and in addition, the assist torque is a nonlinear function of the driver's torque input at the steering wheel.

The key components of the hydraulic power assist system are the hydraulic pump, the rotary spool valve, and the rack piston. The rotary spool valve consists of the torsion bar, inner spool, and outer sleeve. When the driver applies a torque to the steering wheel, the torsion bar twists, and the inner spool rotates with respect to the outer sleeve. This rotation opens metering orifices that increase the flow of hydraulic fluid to one side of the rack piston while restricting flow to the other side of the piston. The differential pressure inside the cylinder pushes the piston to one side or the other depending on the direction of steering. The piston is connected to the steering rack, so hydraulic pressure directly translates to steering effort.

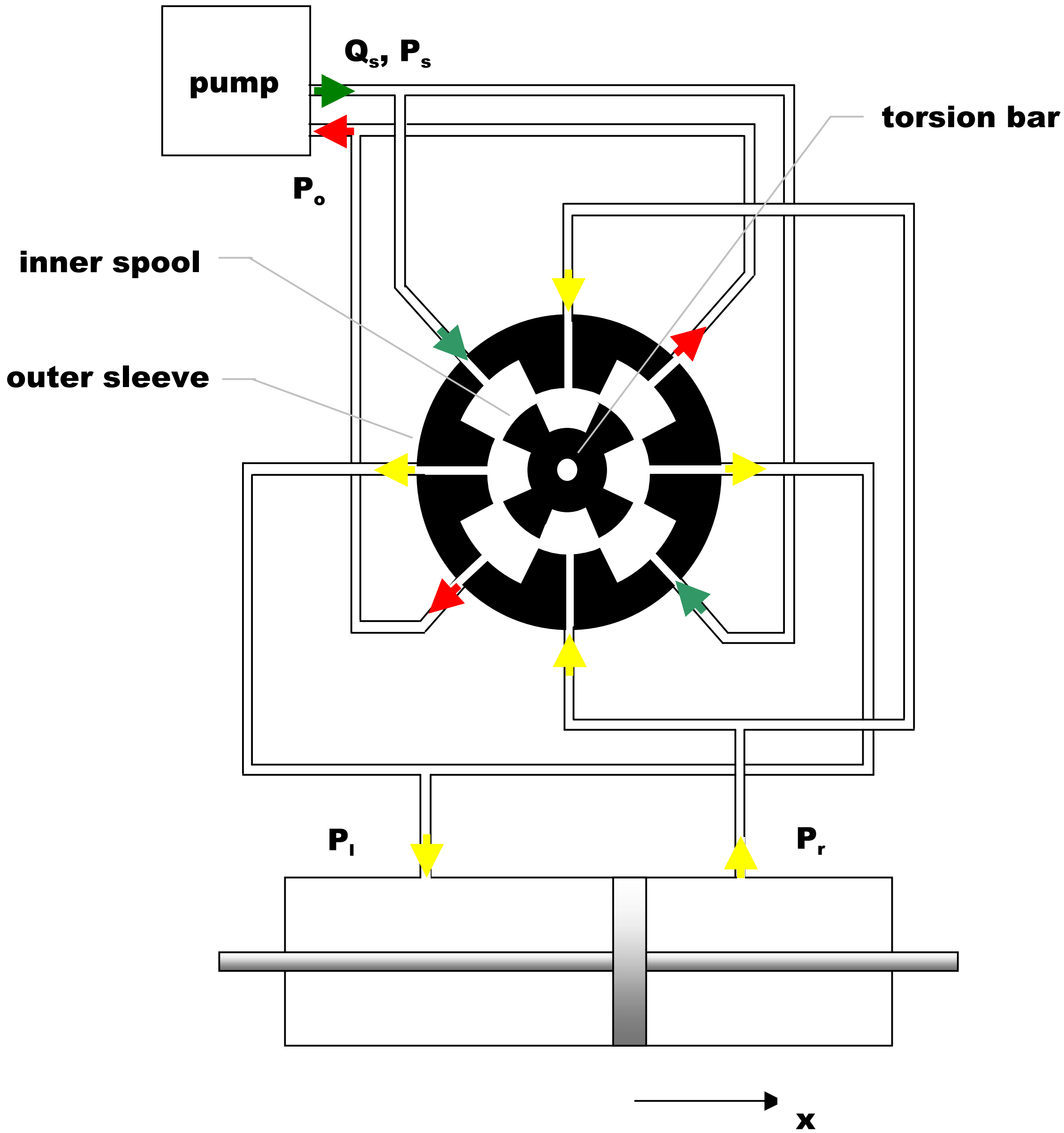

Figure 4: Hydraulic power assist schematic.

Flow through an orifice is described by the following equation

$$Q = AC_d\sqrt{\frac{2\Delta P}{\rho}}$$

where A is the cross-sectional area of the orifice, C_d is the flow coefficient, ρ is the fluid density, and ΔP is the differential pressure across the orifice. By applying this equation to the metering orifices of the rotary spool valve and further applying mass conservation to the entire hydraulic system as in [2],

$$Q_s - A_1 C_d \sqrt{\frac{2}{\rho}} \sqrt{|P_s - P_r|} - A_2 C_d \sqrt{\frac{2}{\rho}} \sqrt{|P_s - P_l|} = \frac{V_s}{\beta} \dot{P}_s$$

$$A_1 C_d \sqrt{\frac{2}{\rho}} \sqrt{|P_s - P_r|} - A_2 C_d \sqrt{\frac{2}{\rho}} \sqrt{|P_r - P_o|} - A_p \dot{x}_r = \frac{A_p \left(\frac{L}{2} + x_r \right)}{\beta} \dot{P}_r$$

$$A_2 C_d \sqrt{\frac{2}{\rho}} \sqrt{|P_s - P_l|} - A_1 C_d \sqrt{\frac{2}{\rho}} \sqrt{|P_l - P_o|} + A_p \dot{x}_r = \frac{A_p \left(\frac{L}{2} - x_r \right)}{\beta} \dot{P}_l$$

where A_1 and A_2 are functions of the torsion bar angle. Note that rack displacement is steering angle multiplied by the steering arm length.

$$x_r = l\theta_w$$

Thus, steering torque is net force on the piston due to pressure multiplied the steering arm length.

$$\tau_p = lA_p \left(P_l - P_r \right)$$

symbol	description
P_s	pump pressure (N/ m^2)
P_r	right cylinder pressure (N/ m^2)
P_l	right cylinder pressure (N/ m^2)
A_1	metering orifice area (m^2)
A_2	metering orifice area (m^2)
x_r	rack displacement (m)

Table 4: Hydraulic system variables.

symbol	description	value
l	steering arm length (m)	0.1
P_o	return pressure (N/ m^2)	0
Q_s	pump flow rate (m^3/s)	0.0002
A_p	piston area (m^2)	0.001
L	cylinder length (m)	0.15
C_d	orifice flow coefficient	0.6
ρ	fluid density (kg/ m^3)	825

V_s	fluid volume (m^3)	$8.2 \cdot 10^{-6}$
β	fluid bulk modulus (N/m^2)	$5.5 \cdot 10^{8}$

Table 5: Hydraulic system parameters.

Steering system dynamics

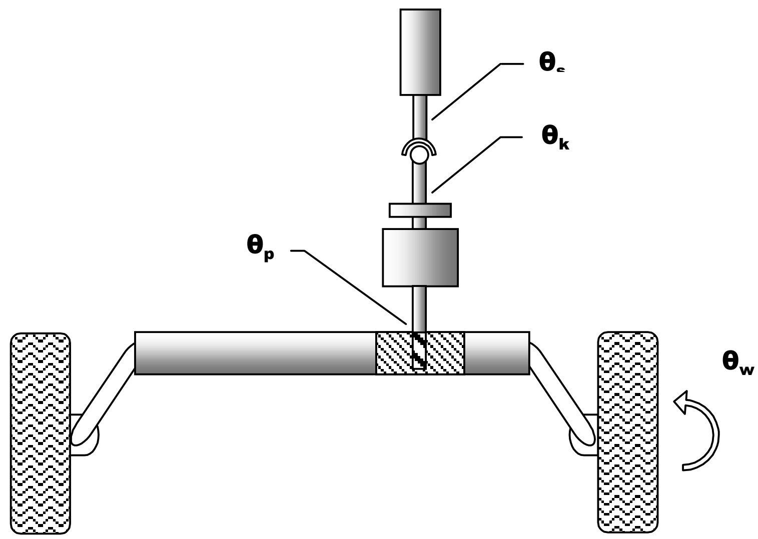

Figure 5: Steering system schematic.

The full equations of motion for the two degree of freedom model are as follows

$$J_w \ddot{\theta}_w = -c_w \dot{\theta}_w + \tau_k r_s + \tau_p - \tau_a$$

$$J_s \ddot{\theta}_s = -c_s \dot{\theta}_s + \tau_s - \tau_k$$

where τ_a is the tire aligning moment calculated from the tire model. The torsion bar torque is defined as

$$\tau_k = k_t \left(\theta_k - \theta_p \right)$$

with

$$\theta_p = r_s \theta_w$$

relating the pinion angle and front wheel steer angle.

symbol	description
θ_k	steering shaft angle (rad)
τ_k	torsion bar torque (N m)
θ_p	pinion angle (rad)
θ_w	road wheel angle (rad)

Table 6: Steering system variables.

symbol	description	value
r_s	steering ratio	16.1
k_t	torsion bar stiffness (N m/rad)	100
J_s	lumped inertia of rotor and coupling shaft (N m s^2/rad)	0.0035
c_s	rotor damping (N m s/rad)	0.1
J_w	lumped inertia of steering rack and wheels about steering axis (N m s^2/rad)	10
c_w	rack and tire damping about steering axis (N m s/rad)	300

Table 7: Steering system parameters.

References

[1] Liu, S. Chang. Force feedback in a stationary driving simulator. *IEEE* 1995, 1711-1716.

[2] Proca, A, Keyhani. Identification of power steering system dynamic models. *Mechatronics* 1998, 255-270.

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Addendum

Universal joint shaft displacement and torque.

Metering orifice area versus torsion bar angle.

Assist torque versus input torque.