# Steer-by-Wire for Vehicle State Estimation and Control

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In order to implement electronically variable dynamics for vehicle handling, the control system requires an accurate knowledge of the vehicle states as well as a means of actuation to precisely influence the vehicle's motion. Steer-by-wire capability conveniently addresses both of these requirements. This paper first presents an approach to estimating vehicle sideslip angle using steering torque information. This method is especially suited to vehicles equipped with steer-by-wire since the steering torque can easily be determined from the current applied to the steering motor. By combining a linear vehicle model with the steering system model, a simple observer may be devised to estimate sideslip when yaw rate and steering angle are measured. Based on this estimate of sideslip angle, a type of state feedback control has been developed to effectively alter the handling characteristics of a vehicle through active steering intervention. Both the observer and its application to vehicle handling modification are demonstrated on an experimental vehicle equipped with steer-by-wire capability.

Topics / Vehicle Dynamics Control, Steering Assistance and Control

#### 1. INTRODUCTION

While steer-by-wire offers unprecedented flexibility in shaping a vehicle's dynamic handling behavior [2, 7], this promise can only be realized with accurate feedback of the vehicle states [5]. Unlike yaw rate, which is readily measured in production vehicles with inexpensive sensors, sideslip angle must be estimated by more sophisticated means. Electronic stability control (ESC) systems currently available on production cars typically derive this value from integration of inertial sensors, but this estimation method is prone to uncertainty and errors [1, 3, 8]. For example, direct integration can accumulate sensor errors and unwanted measurements from road grade and bank angle. An alternative estimation scheme overcomes some of these drawbacks by supplementing integration of inertial sensors with Global Positioning System (GPS) measurements [6]. However, during periods of GPS signal loss, which frequently occur in urban driving environments, integration errors can still accumulate and lead to faulty estimates.

Fortuitously, steer-by-wire provides a ready solution to the problem of sideslip angle estimation. A complete knowledge of steering torque can be determined from the current applied to the system's steering actuator. Through the tire self-aligning moment, steering torque can be directly related to the front tire lateral forces and therefore the wheel



Fig. 1: Experimental steer-by-wire vehicle.

slip angles. This paper develops a two-part observer structure based on linear models of the vehicle and tire behavior to estimate the vehicle states from measurements of steering angle and yaw rate. First, a disturbance observer based on the steering system model estimates the tire aligning moment; this estimate then becomes the measurement part of a vehicle state observer for sideslip and yaw rate. This approach to sideslip estimation also translates to vehicles equipped with electric power steering, since steering torque information can be obtained from the power steering system.

The latter part of the paper applies the state estimation scheme to a physically motivated approach for full state feedback control of an actively steered by-wire vehicle. Experimental results clearly show the change in handling behavior achieved with this type of steering control: the outcome is exactly equivalent to changing cornering stiffness of the front tires. This "virtual tire change" results in a modification of the fundamental handling characteristics of the vehicle, i.e. from oversteering to understeering. In addition to matching handling behavior to driver preference, this modification method is able to successfully counteract handling differences caused by shifts in weight distribution as shown in [10].

#### 2. STEER-BY-WIRE SYSTEM

The vehicle considered in this study is a production model 1997 Chevrolet Corvette that has been converted to steer-by-wire (Fig. 1). The stock steering gear is a rack and pinion configuration with hydraulic power assist. The steer-by-wire conversion (Fig. 2) makes use of all the stock components except for the intermediate steering shaft, which is replaced by a brushless DC servomotor actuator to provide steering torque in place of the handwheel. Two rotary position sensors—one on the steering column and the other on the pinion—provide absolute measurements of both angles. The hydraulic power assist unit in the test vehicle is retained as part of the steer-by-wire system. The incorporation of power assist eliminates the need for extensive modifications to the existing steering system and allows the use of a much smaller actuator since the assist unit provides a majority of the steering effort.

The steering actuator, which consists of a motor and gearhead combination controlled by a serve amplifier, was selected based on the maximum torque and speed necessary to steer the vehicle under typical driving conditions including moderate emergency maneuvers. The steer-by-wire control system, developed in [10], determines the current,  $i_M$ , required by the steering servomotor to follow the driver's steering commands.



Fig. 2: Conventional steering system converted to steer-by-wire.



Fig. 3: Steering system dynamics.

### 3. STEERING SYSTEM MODEL

The steer-by-wire system shown in Fig. 3 is described by the following differential equation:

$$J_w\delta + b_w\delta + \tau_f + \tau_a = r_s r_p \tau_M \tag{1}$$

where  $J_w$  and  $b_w$  are the moment of inertia and damping of the steering system at the road wheels and  $\tau_f$  represents Coulomb friction. Furthermore,  $r_s$  is the steering ratio, and  $r_p$  is the torque magnification factor of the power steering system, here approximated by a constant.  $\tau_M$  is the steering actuator torque, which can be written in terms of motor constant,  $k_M$ , motor current,  $i_M$ , motor efficiency,  $\eta$ , and gearhead ratio,  $r_g$ :

$$\tau_M = k_M i_M r_g \eta \tag{2}$$

The aligning moment,  $\tau_a$ , is a function of the steering geometry, particularly caster angle, and the manner in which the tire deforms to generate lateral forces. In Fig. 4,  $F_{y,f}$  is the lateral force acting on the tire,  $\alpha_f$  is the tire slip angle,  $t_p$  is the pneumatic trail, the distance between the resultant point of application of lateral force and the center of the tire,  $t_m$  is the mechanical trail, the distance between the tire center and the steering axis, and U is the velocity of the tire at its center. The total aligning moment is given by

$$\tau_a = F_{y,f}(t_p + t_m) \tag{3}$$

where  $t_p$  and  $t_m$  are only approximately known. Rewriting Eqn. (1) in state space form yields:

$$\dot{x}_1 = A_1 x_1 + B_{1,1} u_1 + B_{1,2} \tau_a \tag{4}$$

where



Fig. 4: Generation of aligning moment.



Fig. 5: Bicycle model.

$$A_{1} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b_{w}}{J_{w}} \end{bmatrix}$$
$$B_{1,1} = \begin{bmatrix} 0 & 0 \\ \frac{r_{s}r_{p}}{J_{w}} & -\frac{1}{J_{w}} \end{bmatrix}$$
$$u_{1} = \begin{bmatrix} \tau_{M} & \tau_{f} \end{bmatrix}^{T}$$
$$B_{1,2} = \begin{bmatrix} 0 \\ -\frac{1}{J_{w}} \end{bmatrix}$$

and the aligning moment,  $\tau_a$ , is treated as an external input to the steering system. The resisting torque,  $\tau_f$ , due to friction is treated as an input:

$$\tau_f = F_w sgn(\dot{\delta}) \tag{5}$$

where the Coulomb friction constant,  $F_w$ , has been identified along with the inertia and damping constants.

## 4. LINEAR VEHICLE MODEL

A vehicle's handling dynamics in the horizontal plane are represented here by the single track, or bicycle model with states of sideslip angle,  $\beta$ , at the center of gravity (CG) and yaw rate, r. In Fig. 5,  $\delta$ is the steering angle,  $u_x$  and  $u_y$  are the longitudinal and lateral components of the CG velocity,  $F_{y,f}$  and  $F_{y,r}$  are the lateral tire forces front and rear, respectively, and  $\alpha_f$  and  $\alpha_r$  are the tire slip angles. Assuming constant longitudinal velocity  $u_x = V$ , the state equation for the bicycle model can be written as:

 $\dot{x}_2 = A_2 x_2 + B_2 \delta$ 

where

$$\begin{aligned} x_2 &= \begin{bmatrix} \beta & r \end{bmatrix}^T \\ A_2 &= \begin{bmatrix} -\frac{C_{\alpha,0}}{mV} & -1 + \frac{C_{\alpha,1}}{mV^2} \\ \frac{C_{\alpha,1}}{T_z} & -\frac{C_{\alpha,2}}{T_zV} \end{bmatrix} \\ B_2 &= \begin{bmatrix} \frac{C_{\alpha,f}}{mV} \\ \frac{C_{\alpha,fa}}{T_z} \end{bmatrix} \end{aligned}$$

and to consolidate notation,

$$C_{\alpha,0} = C_{\alpha,f} + C_{\alpha,r}$$
  

$$C_{\alpha,1} = C_{\alpha,r}b - C_{\alpha,f}a$$
  

$$C_{\alpha,2} = C_{\alpha,f}a^2 + C_{\alpha,r}b^2$$

where a and b are the distance of the front and rear axle from the CG, and  $C_{\alpha,f}$  and  $C_{\alpha,r}$  are the composite front and rear cornering stiffness. Sideslip angle is defined by either

$$\beta = \arctan\left(\frac{u_y}{u_x}\right) \tag{7}$$

or the difference between the vehicle's forward orientation,  $\psi$ , and the direction of the velocity,  $\gamma$ .

$$\beta = \gamma - \psi \tag{8}$$

## 5. VEHICLE STATE ESTIMATION US-ING STEERING TORQUE

#### 5.1 Steering disturbance observer

When looking at the two state linear vehicle model described above, one might consider designing a simple state observer based on measurement of yaw rate alone. Unfortunately, there is one instance in which the sideslip angle is unobservable through yaw rate: the neutral steering case  $(C_{\alpha,r}b - C_{\alpha,f}a)$ equals zero). Therefore, an observer based on yaw rate alone is impractical as the vehicle handling characteristics approach the neutral steering configuration. One way to estimate sideslip in this situation is to first estimate the aligning moment by applying a disturbance observer to the steering system model described by Eqn. (4). The aligning moment estimate then becomes a measurement for the state estimator based on the vehicle model given by Eqn. (6). A disturbance observer structure for the steering system is simply constructed by appending the disturbance,  $\tau_a$ , to the state vector,  $x_1$ , and augmenting the corresponding rows in the state matrices with zeroes:

$$\dot{z}_1 = F_1 z_1 + G_1 u_1 \tag{9}$$

where

(6)

$$z_{1} = \begin{bmatrix} \delta & \dot{\delta} & \tau_{a} \end{bmatrix}^{T}$$

$$F_{1} = \begin{bmatrix} A_{1} & B_{1,2} \\ 0 & 0 \end{bmatrix}$$

$$G_{1} = \begin{bmatrix} B_{1,1} \\ 0 \end{bmatrix}$$

The available measurement,  $y_1$ , is the steering angle,  $\delta$ :

$$y_1 = \delta = C_1 z_1 \tag{10}$$

where

$$C_1 = \left[ \begin{array}{ccc} 1 & 0 & 0 \end{array} \right]$$

The disturbance observer is given by:

$$\hat{z}_1 = (F_1 - L_1 C_1) \hat{z}_1 + G_1 u_1 + L_1 y_1$$
 (11)

and the corresponding error dynamics are:

$$\dot{\tilde{z}}_1 = (F_1 - L_1 C_1) \tilde{z}_1$$
 (12)

where the estimation error is

$$\tilde{z}_1 = z_1 - \hat{z}_1$$

This formulation of the disturbance observer is a technical simplification which assumes the derivative of disturbance torque,  $\dot{\tau}_a$ , is zero. In other words, it assumes the disturbance is varying slowly and independent of the steering system dynamics. In reality, as is evident from Eqn. (3), the derivative of the disturbance does depend on the steering rate as well the dynamics of the vehicle. Making the assumption that  $\dot{\tau}_a$  equals zero, however, results in a close approximation of disturbance torque and is similar to the approach taken in [9].

#### 5.2 Vehicle state observer

Now the standard observer structure is applied to the vehicle model described by Eqn. (6):

$$\dot{\hat{x}}_2 = A_2 \hat{x}_2 + B_2 u_2 + T_2 (y_2 - \hat{y}_2)$$
(13)

The vector,  $\hat{x}_2$ , contains the states to be estimated and  $y_2$  is the vector of "measurements"—in this case, yaw rate and the aligning moment estimate obtained from the disturbance observer. Note that the aligning moment given by Eqn. (3) can be expressed in terms of the vehicle states so that

$$y_2 = \begin{bmatrix} r & \tau_a \end{bmatrix}^T = C_2 x_2 + D_2 \delta \tag{14}$$

where

$$C_2 = \begin{bmatrix} 0 & 1\\ -(t_p + t_m)C_{\alpha,f} & -\frac{a(t_p + t_m)C_{\alpha,f}}{V} \end{bmatrix}$$
$$D_2 = \begin{bmatrix} 0\\ (t_p + t_m)C_{\alpha,f} \end{bmatrix}$$

While Eqn. (6) is unobservable in the neutral steering case when yaw rate, r, is the sole measurement, the addition of aligning moment,  $\tau_a$ , to the measurement vector means that the system given by Eqn. (6) and Eqn. (14) will always be observable. The observer in Eqn. (13) can be rewritten:

$$\dot{\hat{x}}_2 = (A_2 - T_2 C_2) \hat{x}_2 + (B_2 - T_2 D_2) \delta + T_2 y_2$$
 (15)

As before, the estimator gain matrix,  $T_2$ , is chosen so that the matrix  $A_2 - T_2C_2$  has stable eigenvalues and the error dynamics are significantly faster than the system dynamics. The error dynamics here are given by:

$$\tilde{x}_2 = (A_2 - T_2 C_2) \tilde{x}_2$$
(16)

where the estimation error is

$$\tilde{x}_2 = x_2 - \hat{x}_2$$

#### 6. CLOSED LOOP VEHICLE CONTROL

#### 6.1 Handling modification

The basis for handling modification of an actively steered by-wire vehicle is to apply these estimated states to closed loop control of the vehicle dynamics. The full state feedback control law for an active steering vehicle is given by

$$\delta = K_r r + K_\beta \beta + K_d \delta_d \tag{17}$$

where  $\delta_d$  is the driver commanded steer angle and  $\delta$  is the augmented angle. A physically intuitive way to modify a vehicle's handling characteristics is to define a target front cornering stiffness as

$$\hat{C}_{\alpha,f} = C_{\alpha,f}(1+\eta) \tag{18}$$

and the state feedback gains as

$$K_{\beta} = -\eta \qquad K_r = -\frac{a}{V}\eta \qquad K_d = (1+\eta) \qquad (19)$$

where  $\eta$  is the desired fractional change in the original front cornering stiffness  $C_{\alpha,f}$ . Substituting the feedback law, Eqn. (17), into Eqn. (6) yields a state space equation of the same form as Eqn. (6) but with the new cornering stiffness  $\hat{C}_{\alpha,f}$ :

$$\dot{x}_2 = \hat{A}_2 x_2 + \hat{B}_2 \delta \tag{20}$$

where

$$\begin{aligned} x_2 &= \begin{bmatrix} \beta & r \end{bmatrix}^T \\ \hat{A}_2 &= \begin{bmatrix} -\frac{\hat{C}_{\alpha,0}}{mV} & -1 + \frac{\hat{C}_{\alpha,1}}{mV^2} \\ \frac{\hat{C}_{\alpha,1}}{I_z} & -\frac{\hat{C}_{\alpha,2}}{I_zV} \end{bmatrix} \\ \hat{B}_2 &= \begin{bmatrix} \frac{\hat{C}_{\alpha,f}}{mV} \\ \frac{\hat{C}_{\alpha,fa}}{I_z} \end{bmatrix} \end{aligned}$$

and to consolidate notation

$$\hat{C}_{\alpha,0} = \hat{C}_{\alpha,f} + C_{\alpha,r} \hat{C}_{\alpha,1} = C_{\alpha,r}b - \hat{C}_{\alpha,f}a \hat{C}_{\alpha,2} = \hat{C}_{\alpha,f}a^2 + C_{\alpha,r}b^2$$

Since a vehicle's handling characteristics are heavily influenced by tire cornering stiffness, the effect of this modification is to make the vehicle either more oversteering or understeering depending on the sign of  $\eta$ . Of course, there are many other ways to apply full state feedback, but the physical motivation behind cornering stiffness adjustment makes clear through the bicycle model exactly how the handling characteristics have been modified. In fact, the effect of this modification is exactly equivalent to altering a vehicle's handling behavior by changing the tires as is often done—in automotive racing terms during a pit stop.



Fig. 6: Comparison between estimated yaw rate, INS measurement, and bicycle model simulation with normal cornering stiffness.



Fig. 7: Comparison between estimated sideslip angle, GPS measurement, and bicycle model simulation with normal cornering stiffness.

#### 6.2 Experimental results

As developed thus far in the paper, all of the components necessary for physical implementation of closed loop vehicle dynamics control are now in place: 1) accurate state estimates are available from the observer described in the previous section, 2) a means of precise vehicle control is provided by the steer-by-wire system in the test vehicle, and 3) a full state feedback control law has been devised to virtually and fundamentally alter a vehicle's handling characteristics. The experimental results presented below are based on the following test procedure: the vehicle is accelerated from standstill in a straight line; once it reaches a steady speed of 13.4m/s (30mi/hr), the onboard computer begins to generate a sinusoidal steering command of constant amplitude and frequency (equivalent to a



Fig. 8: Comparison between lateral acceleration with normal and effectively reduced front cornering stiffness.

driver's input at the steering wheel). For the first test run, the vehicle is driven in the unmodified mode (no state feedback) such that the road wheel angle corresponds directly to command angle scaled by the steering ratio. In the plots of yaw rate and sideslip angle (Figs. 6 and 7) from this test, the estimated values from the state observer are compared with both GPS/INS measurement and bicycle model simulation. Yaw rate estimated from the observer matches the GPS/INS numbers almost exactly since it is the "measured" state by which the observer determines the unmeasurable state of sideslip angle. More importantly, sideslip angle estimated from the observer also closely follows GPS measurement and model prediction.

Next, the same test is repeated with the effective front cornering stiffness reduced 50% by setting the parameter  $\eta$  to -0.5. The resulting difference in handling behavior is evident when comparing yaw rate and sideslip angle (Figs. 9 and 10) to the nominal case. As expected, the modified handling exhibits lower peak yaw rate and sideslip values since the effect of reduced front cornering stiffness is more pronounced understeering behavior.

## 7. CONCLUSION

As steering torque information becomes more common in automotive steering systems—in the form of either electric power steering or steer-bywire—a useful connection can be drawn between forces and vehicle motion: the knowledge of forces acting on the steering system through the tires in turn provides information on the motion of the vehicle itself. Like GPS-based estimation, vehicle state estimation using steering torque is not subject to the problems of error accumulation from inertial sensor integration. Unlike GPS, however, the signal is



Fig. 9: Comparison between yaw rate with normal and effectively reduced front cornering stiffness.



Fig. 10: Comparison between sideslip angle with normal and effectively reduced front cornering stiffness.

never lost, and no extra and expensive equipment is necessary if a vehicle is equipped with electric power steering or, in the near future, steer-by-wire technology.

An observer structure based on linear models of the vehicle and steering system dynamics has been developed to take advantage of this additional measurement. As demonstrated in the experimental work, the combination of readily available measurements from steering torque, steering angle, and yaw rate sensors generates a sideslip angle estimate comparable to that obtained from highly accurate measurements by a sophisticated GPS/INS system. Furthermore, the sideslip estimation has been successfully implemented as a feedback signal for closed loop vehicle control. This approach has many practical implications for the next generation of fully integrated automotive stability control systems, since all of the measurement devices necessary for precise vehicle control already exist and have been inexpensively implemented on production cars. Future work will investigate how to extend the ability of the observer to predict vehicle motion beyond the linear range of handling behavior by, for example, continuously adapting tire cornering stiffness to the current driving situation [4].

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