Vehicle State Estimation Using Steering Torque

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Abstract—This paper presents a new approach to estimating vehicle sideslip using steering torque information. This method is especially suited to vehicles equipped with steer-by-wire systems since the steering torque can easily be determined from the current applied to the steering motor. By combining a linear vehicle model with the steering system model, a simple observer may be devised to estimate sideslip when yaw rate and steering angle are measured. The observer is validated on a test vehicle equipped with a steer-by-wire system.

I. INTRODUCTION

The effectiveness of a vehicle dynamic stability control system relies on accurate knowledge of the vehicle states, particularly yaw rate and sideslip angle. While yaw rate is readily measured in production vehicles with inexpensive sensors, sideslip angle must be estimated by more sophisticated means. Stability systems currently available on production cars typically derive this value from integration of inertial sensors, but this estimation method is prone to uncertainty and errors [1], [2], [3]. For example, direct integration can accumulate sensor errors and unwanted measurements from road grade and bank angle.

An alternative estimation scheme overcomes some of these drawbacks by supplementing integration of inertial sensors with Global Positioning System (GPS) measurements [4]. Absolute GPS heading and velocity measurements eliminate the errors from inertial navigation system (INS) integration; conversely, INS sensors complement the GPS measurements by providing higher update rate estimates of the vehicle states. However, during periods of GPS signal loss, which frequently occur in urban driving environments, integration errors can still accumulate and lead to faulty estimates.

The growing presence of electric power steering systems in production vehicles introduces yet another absolute measurement—steering torque—from which vehicle sideslip angle may be estimated. Through the tire self-aligning moment, steering torque is directly related to the lateral front tire forces, which in turn relate to the tire slip angles and therefore the vehicle states. This paper develops two observer structures based on linear models of the vehicle and tire behavior to estimate the vehicle states from measurements of steering angle and yaw rate.
The incorporation of power assist eliminates the need for extensive modifications to the existing steering system and allows the use of a much smaller actuator since the assist unit provides a majority of the steering effort.

The steering actuator, which consists of a motor and gearhead combination controlled by a servo amplifier, was selected based on the maximum torque and speed necessary to steer the vehicle under typical driving conditions including moderate emergency maneuvers. The steer-by-wire control system, developed in [7], determines the current, \( i_M \), required by the steering servomotor to follow the driver's steering commands.

### III. STEERING SYSTEM MODEL

The steering system shown in Fig. 3 is described by the following differential equation:

\[
J_w \ddot{\delta} + b_w \dot{\delta} + \tau_f + \tau_a = r_s r_p \tau_M \tag{1}
\]

where \( J_w \) and \( b_w \) are the moment of inertia and damping of the steering system at the road wheels and \( \tau_f \) represents Coulomb friction. Furthermore, \( r_s \) is the steering ratio, and \( r_p \) is the torque magnification factor of the power steering system, here approximated by a constant. \( \tau_M \) is the steering actuator torque, which can be written in terms of motor constant, \( k_M \), motor current, \( i_M \), motor efficiency, \( \eta \), and gearhead ratio, \( r_g \):

\[
\tau_M = k_M i_M r_g \eta \tag{2}
\]

The tire self-aligning moment, \( \tau_a \), is a function of the steering geometry, particularly caster angle, and the manner in which the tire deforms to generate lateral forces. In Fig. 4, \( F_{y,f} \) is the lateral force acting on the tire, \( \alpha_f \) is the tire slip angle, \( t_p \) is the pneumatic trail, the distance between the resultant point of application of lateral force and the center of the tire, \( t_m \) is the mechanical trail, the distance between the tire center and the steering axis, and \( U \) is the velocity of the tire at its center. The total aligning moment is given by

\[
\tau_a = F_{y,f} (t_p + t_m) \tag{3}
\]

where \( t_p \) and \( t_m \) are approximately known. Rewriting (1) in state space form yields:

\[
\dot{x}_1 = A_1 x_1 + B_{1,1} u_1 + B_{1,2} \tau_a \tag{4}
\]

where

\[
x_1 = \begin{bmatrix} \delta \\ \dot{\delta} \end{bmatrix}^T
\]

\[
A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -b_w/J_w \end{bmatrix}
\]

\[
B_{1,1} = \begin{bmatrix} 0 \\ \frac{r_s r_p}{J_w} - \frac{1}{J_w} \end{bmatrix}
\]

\[
u_1 = \begin{bmatrix} \tau_M \\ \tau_f \end{bmatrix}^T
\]

\[
B_{1,2} = \begin{bmatrix} 0 \\ -\frac{1}{J_w} \end{bmatrix}
\]

and the aligning moment, \( \tau_a \), is treated as an external input to the steering system. The resisting torque, \( \tau_f \), due to friction is treated as an input:

\[
\tau_f = F_w \text{sgn}(\dot{\delta}) \tag{5}
\]

where the Coulomb friction constant, \( F_w \), has been identified along with the inertia and damping constants.

### IV. LINEAR VEHICLE MODEL

A vehicle's handling dynamics in the horizontal plane are represented here by the single track, or bicycle model with states of sideslip angle, \( \beta \), at the center of gravity (CG) and yaw rate, \( \dot{\gamma} \). In Fig. 5, \( \delta \) is the steering angle, \( u_x \) and \( u_y \) are the longitudinal and lateral components of the CG velocity, \( F_{y,f} \) and \( F_{y,r} \) are the lateral tire forces front and rear, respectively, and \( \alpha_f \) and \( \alpha_r \) are the tire slip angles. Derivation of the equations of motion for the bicycle model follows from the force and moment balance:

\[
m a_y = F_{y,f} \cos \delta + F_{y,r} \tag{6}
\]

\[
I_z \ddot{\gamma} = a F_{y,f} \cos \delta - b F_{y,r}
\]

Fig. 2. Conventional steering system converted to steer-by-wire.

Fig. 3. Steering system dynamics.

Fig. 4. Generation of aligning moment.
Taking small angle approximations, slip angle can be written in terms of the CG. In the linear region of tire operation—typically slip velocity, $u$, by:

$$
I_x \text{ is the moment of inertia of the vehicle about its yaw axis, } m \text{ is the vehicle mass, } a \text{ and } b \text{ are distance of the front and rear axles from the CG, and } a_y \text{ is lateral acceleration at the CG. In the linear region of tire operation—typically slip angles of four degrees or less—lateral force at the front and rear is related to slip angle by the total cornering stiffness coefficient of the front and rear tires:}

$$
F_{y,f} = -C_{\alpha,f} \alpha_f \\
F_{y,r} = -C_{\alpha,r} \alpha_r
$$

Taking small angle approximations, slip angle can be written in terms of $u_x$, $u_y$, and $r$:

$$
\alpha_f = \frac{u_y + ar}{u_x} - \delta \\
\alpha_r = \frac{u_y - br}{u_x}
$$

Assuming constant longitudinal velocity $u_x = V$, the state equation for the bicycle model can be written as:

$$
\dot{x}_2 = A_2 x_2 + B_2 \delta
$$

with states of vehicle sideslip angle, $\beta$, yaw rate, $\psi$, and the direction of the velocity, $\gamma$.

$$
\beta = \gamma - \psi
$$

V. CONVENTIONAL OBSERVER

When looking at the two state linear vehicle model described above, one might consider designing a simple state observer based on measurement of yaw rate alone. Unfortunately, there is one instance in which the sideslip angle is unobservable through yaw rate: the neutral steering case $(C_{\alpha,r} b - C_{\alpha,f} a$ equals zero). Therefore, an observer based on yaw rate alone is impractical as the vehicle handling characteristics approach the neutral steering configuration. One way to estimate sideslip in this situation is to combine the linear vehicle model with the steering system model.

The aligning moment term in the steering system equation can be expressed as a function of the vehicle states and the linear vehicle model with the steering system model.

Now combining (4) and (9) yields the following state space model:

$$
x_3 = A_3 x_3 + B_3 u_3
$$

where

$$
x_3 = \begin{bmatrix} \beta & r & \delta & \dot{\delta} \end{bmatrix}^T \\
A_3 = \begin{bmatrix} -\frac{C_{\alpha,0}}{mV} & -1 + \frac{C_{\alpha,1}}{mV^2} & \frac{C_{\alpha,f}}{mV} & 0 \\
\frac{C_{\alpha,1}}{I_x} & -\frac{C_{\alpha,2}}{I_x} & \frac{C_{\alpha,r}}{I_x} & 0 \\
0 & 0 & 0 & 1 \\
r_x r_p k m r_s \eta & -1 & \frac{1}{J_w} \\
0 & 0 & 0 & 0 \\
r_0 & 0 & 0 & 0 \\
\end{bmatrix} \\
B_3 = \begin{bmatrix} 0 \\
0 \\
0 \\
0 -1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} \\
u_3 = \begin{bmatrix} \dot{\tau}_f \end{bmatrix}^T
$$

and

$$C_{\alpha,3} = (t_p + t_m) C_{\alpha,f}$$

with states of vehicle sideslip angle, $\beta$, yaw rate, $r$, steering angle, $\delta$, and steering rate, $\dot{\delta}$. Note that after incorporating the steering system dynamics this system is now observable in the neutral steering case. The standard observer structure is given by:

$$
\dot{\hat{x}}_3 = A_3 \hat{x}_3 + B_3 u_3 + T_3(y_4 - C_3 \hat{x}_3)
$$

The vector, $\hat{x}_3$, contains the estimated states and $y_3$ is the vector of measurements—yaw rate and steering angle—directly available from vehicle sensors:

$$
y_3 = \begin{bmatrix} r & \delta \end{bmatrix}^T = C_3 x_3
$$
where
\[ C_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]

The estimator gain matrix, \( C_3 \), is chosen so that the matrix \( A_3 - T_3 C_3 \) has eigenvalues located at least ten times farther to the left than the system eigenvalues in the complex frequency plane. The sampling rate of the measurements should be yet another order of magnitude faster than the estimation error dynamics, which are given by:
\[ \dot{x}_3 = (A_3 - T_3 C_3) \hat{x}_3 \]  
(17)

where the estimation error is
\[ \hat{x}_3 = x_3 - \hat{x}_3 \]

When \( T_3 \) is selected so that \( A_3 - T_3 C_3 \) has stable eigenvalues, the error dynamics approach zero exponentially.

VI. DISTURBANCE OBSERVER

As an alternative, one can first estimate the aligning moment by applying a disturbance observer to the steering system model described by (4). The aligning moment estimate then becomes a measurement for the state estimator based on the vehicle model given by (9). A disturbance observer structure for the steering system is simply constructed by appending the disturbance, \( \tau_a \), to the state vector, \( x_1 \), and augmenting the corresponding rows in the state matrices with zeroes:
\[ \dot{z}_1 = F_1 \hat{z}_1 + G_1 u_1 \]  
(18)

where
\[ z_1 = \begin{bmatrix} x_1^T \\ x_2 \end{bmatrix}, \quad F_1 = \begin{bmatrix} A_1 & B_{1,2} \\ 0 & 0 \end{bmatrix}, \quad G_1 = \begin{bmatrix} B_{1,1} \\ 0 \end{bmatrix} \]

The available measurement, \( y_1 \), is the steering angle, \( \delta \):
\[ y_1 = \delta = C_1 z_1 \]  
(19)

where
\[ C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \]

The disturbance observer is given by:
\[ \ddot{z}_1 = (F_1 - L_1 C_1) \dot{z}_1 + G_1 u_1 + L_1 y_1 \]  
(20)

and the corresponding error dynamics are:
\[ \ddot{\hat{z}}_1 = (F_1 - L_1 C_1) \hat{z}_1 \]  
(21)

where the estimation error is
\[ \hat{z}_1 = z_1 - \dot{\hat{z}}_1 \]

This formulation of the disturbance observer is a technical simplification which assumes the derivative of disturbance torque, \( \dot{\tau}_a \), is zero. In other words, it assumes the disturbance is varying slowly and independent of the steering system dynamics. In reality, as is evident from (3), the derivative of the disturbance does depend on the steering rate as well the dynamics of the vehicle. Making the assumption that \( \dot{\tau}_a \) equals zero, however, results in a close approximation of disturbance torque and is similar to the approach taken in [8].

Now the standard observer structure is applied to (9) only:
\[ \dot{\hat{x}}_2 = A_2 \hat{x}_2 + B_2 u_2 + T_2 (y_2 - \hat{y}_2) \]  
(22)

The vector, \( \hat{x}_2 \), contains the states to be estimated and \( y_2 \) is the vector of "measurements"—in this case, yaw rate and the aligning moment estimate obtained from the disturbance observer. Recall that substituting (8), (9) and (2) into (1) expresses the aligning moment, \( \tau_a \), in terms of the vehicle states, \( \beta \) and \( r \):
\[ y_2 = \begin{bmatrix} r \\ \tau_a \end{bmatrix}^T = C_2 x_2 + D_2 \delta \]  
(23)

where
\[ C_2 = \begin{bmatrix} 0 \\ -(t_p + t_m) C_{a,f} \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ (t_p + t_m) C_{a,f} \end{bmatrix} \]

While (9) is unobservable in the neutral steering case when yaw rate, \( r \), is the sole measurement, the addition of aligning moment, \( \tau_a \), to the measurement vector means that the system given by (9) and (23) will always be observable.

The observer in (22) can be rewritten:
\[ \dot{\hat{x}}_2 = (A_2 - T_2 C_2) \hat{x}_2 + (B_2 - T_2 D_2) \delta + T_2 y_2 \]  
(24)

As before, the estimator gain matrix, \( T_2 \), is chosen so that the matrix \( A_2 - T_2 C_2 \) has stable eigenvalues and the error dynamics are significantly faster than the system dynamics. The error dynamics here are given by:
\[ \ddot{\hat{x}}_2 = (A_2 - T_2 C_2) \hat{x}_2 \]  
(25)

where the estimation error is
\[ \hat{x}_2 = x_2 - \hat{x}_2 \]

VII. EXPERIMENTAL RESULTS

Both types of state observer have been implemented in real-time on the steer-by-wire test vehicle. The state estimates from the observers are simultaneously compared to results from a highly accurate sideslip estimation method based on measurements from a GPS/INS system installed in the test vehicle. Details and validation of the GPS-driven state estimation method can be found in [4]. As a reference, vehicle states calculated from the linear vehicle model—with parameters matched to the test vehicle—are also included in the comparison. The following figures correspond to the same test cycle during which the vehicle
is accelerated from a standing stop to a steady speed of $6.7\, m/s (15\, mi/hr)$ at which time the steering maneuver is initiated.

Figs. 6 and 7 show the state estimates from the four-state observer. As expected, since steering angle and yaw rate are measurements, the estimates of steering angle and yaw rate tend to match the measured values almost exactly. The observed state of primary interest, sideslip angle, correlates well with the GPS/INS sideslip estimate, but the choice of observer gains found to produce a good steering rate estimate fails to filter noise in the sideslip estimate (Fig. 7). This problem illustrates the drawback of the four-state observer: the choice of a single observer gain matrix often compromises estimation performance due to the fact that the steering system dynamics are significantly faster than the vehicle dynamics.

By separating the state estimation into a disturbance observer for the steering system and a two-state observer for the vehicle states, the alternative observer structure facilitates the selection of observer gains appropriate to either set of dynamics. The improvement is clear in the sideslip estimate of Fig. 9 and the steering rate estimate of Fig. 8.

Since the vehicle is operated well within the linear region of its dynamic behavior in these tests, the yaw rate and sideslip predicted by the linear vehicle model follow the estimated values closely. The estimated disturbance torque shown in Fig. 10, however, is not as well predicted by the aligning moment calculated from (3). In modeling the steering system of the test vehicle, several sources of uncertainty exist to cause such discrepancy, among them nonlinear hydraulic power steering characteristics and changes in suspension geometry (toe, camber, and caster angle) due to steering and suspension motions. Fortunately,
many of the physical unknowns associated with test vehicle control can be overcome in a clean-sheet steer-by-wire design. For example, if the suspension system were designed with large and invariant caster angle, the predictable effects of tire mechanical trail would dominate the nonlinear characteristics of the pneumatic trail.

It is important to note that although the imperfect disturbance torque estimate is used as a measurement in the vehicle state observer, the estimates of the vehicle states do not apparently suffer in the same way from these uncertainties. This suggests that two-tiered vehicle state estimation scheme can be made to be fairly robust to parameter uncertainty in the steering system model as long as there is a reliable vehicle model with accurate yaw rate measurement.

VIII. Conclusion

As steering torque information becomes more common in automotive steering systems—in the form of either electric power steering or steer-by-wire—a useful connection can be drawn between forces and vehicle motion: the knowledge of forces acting on the steering system through the tires in turn provides information on the motion of the vehicle itself. Like GPS-based estimation, vehicle state estimation using steering torque is not subject to the problems of error accumulation from inertial sensor integration. Unlike GPS, however, the signal is never lost, and no extra and expensive equipment is necessary if a vehicle is equipped with electric power steering or, in the near future, steer-by-wire technology.

Two observer structures based on linear models of the vehicle and steering system dynamics have been developed to take advantage of this additional measurement. As demonstrated in the experimental work, the combination of readily available measurements from steering torque, steering angle, and yaw rate sensors generates a sideslip angle estimate comparable to that obtained from highly accurate measurements by a sophisticated GPS/INS system. This has many practical implications for the next generation of fully integrated automotive stability control systems, since all of the measurement devices necessary for precise vehicle control already exist and have been inexpensively implemented on production cars.

Furthermore, in a carefully designed steering system, the aligning moment disturbance effectively communicates the tire forces acting on the vehicle—regardless of whether it is being operated in the linear or nonlinear handling region—and thus addresses the limitations of using a linear model-based observer structure. Future work will investigate how to apply this information to extend the ability of the observer to predict vehicle motion throughout the entire range of handling behavior.

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