COMBINING LANEKEEPING AND VEHICLE FOLLOWING WITH HAZARD MAPS

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Abstract

This paper addresses the issues involved with including moving obstacles in a hazard map or potential field framework for driver assistance systems. By treating vehicle following as a combination of a safety distance and a hazard or potential function, common following strategies, such as constant time headway and guaranteed collision avoidance, can be incorporated into this framework without modification. When combining these fields with lateral potential fields for lanekeeping, however, challenges arise due to the natural asymmetry between the longitudinal and lateral velocity of a vehicle. One solution to this is to scale the lateral force associated with the moving vehicle, thus deviating from a strict potential field paradigm for control. This can still be shown to produce a bounded hazard in the absence of driver input, though the entire system energy is no longer guaranteed to decrease.

1 Introduction

As vehicle safety systems expand to encompass functions such as stability control, lanekeeping and collision avoidance, tighter integration of these control tasks is required. To meet this challenge, several paradigms for systems integration have been proposed. Hennessey *et al.* (1995) looked at combining control objectives using the framework of "virtual bumpers". In this approach, the vehicle behaved as if there were imaginary springs and dampers attached to the vehicle's center. While a simple idea intuitively, Hennessey *et al.* (1995) detailed complexities that can arise with bifurcations and Schiller *et al.* (1998*b*) noted that tuning stiffnesses can be a challenge when lanekeeping and collision avoidance objectives were combined.

Reichardt and Schick (1994) proposed controlling an autonomous vehicle in a complex environment by assigning a hazard to different points in the environment. This hazard map then became a potential function with vehicle motion controlled to match the gradient of the map. In this way, the vehicle always moved towards less hazardous environmental states, at least locally. Because of the common notion of hazard, however, no tuning was required, making this approach appealing from the standpoint of weighting safety objectives.

Gerdes and Rossetter (1999) applied this view of hazard maps to driver assistance systems. In this framework, the environment produced a force corresponding to the gradient of the potential function that was applied in concert with the base vehicle dynamics using x-by-wire systems. Under this interpretation of the hazard map, the vehicle was not guaranteed to move in the direction of the gradient since the underlying dynamics were not cancelled. Rather, the overall hazard in the system was guaranteed to be bounded by the initial effective energy (the sum of the real kinetic and artificial potential energy) since the uncancelled dynamics were passive. The worst case scenario was thus for the entire energy of the system to be converted into the artificial potential energy or hazard. The application of this technique was illustrated on a combined lanekeeping and stability control system where all hazards in the environment were considered stationary.

This paper extends the previous work to cover moving hazards in the environment, particularly those of other vehicles in the lane ahead. Although collision avoidance with moving potential fields has been studied extensively, the common approach of letting the potential function approach infinity at the obstacle (Khatib 1986) is not feasible for automotive applications. Saturation of the brakes (and other actuators) makes any guarantees of this nature suspect, particularly since the obstacles are capable at decelerating at rates comparable to that of the controlled vehicle. These issues can be resolved by combining a concept of a safety distance at which the vehicle should follow the lead vehicle with a hazard - or potential function - associated with deviation from that distance. The design and interpretation of such a control structure in light of previous work in vehicle following is discussed in Section 2. For certain cases, the bound of initial energy is shown to be greatly conservative when predicting maximum hazard.

When the following fields are restricted to a single lane and combined with lateral potential fields for lane-keeping, some drawbacks of a strict potential field controller become apparent. Due to the large asymmetry between longitudinal and lateral velocity, a lane change while in the potential field behind a lead vehicle produces an undesirably large increase in lateral velocity. This can be eliminated by exploiting the asymmetry to scale the lateral and longitudinal forces associated with the moving hazard separately. While losing the overall guarantee of decreasing energy, the concept of bounded hazard remains intact. The resulting control structure thus encompasses some of the character of virtual spring stiffnesses in the virtual bumper approach (Hennessey *et al.* 1995, Schiller *et al.* 1998*a*, Schiller *et al.* 1998*b*) while retaining the central concept of hazard (Reichardt and Schick 1994, Gerdes and Rossetter 1999). Suggestions for a more analytical treatment of this scaled potential field controller conclude the paper.

2 Hazard Maps for Following

Vehicle follower laws have been developed by a number of researchers for applications ranging from collision avoidance to intelligent cruise control (Ioannou and Chien 1993) to highway automation (Shladover 1991). Using the approach of Saur (2000), the vehicle location along a straight roadway is given by s and a vehicle in the lane ahead is represented by a position, s_l. Other terms are defined relative to this position. s_d is a safety distance behind the lead car which is set by the following policy and s_{des}

$$s_{des} = s_l - s_d \tag{1}$$

is the desired position for the following car. The spacing error, ϵ_s, is defined relative to the desired position as:

$$\epsilon_s = s - s_{des} \tag{2}$$

The potential field is then defined as some function $V(\epsilon_s)$. In this representation, the safety distance contains the desired spacing dynamics and the potential field represents the hazard associated with following more closely than the follower law dictates.

The general form suggested by Ioannou and Chien (1993) for a safe distance behind the lead vehicle:

$$s_d = \lambda_1 \left(\dot{s}^2 - \dot{s}_l^2\right) + \lambda_2 \dot{s} + \lambda_3 \tag{3}$$

can be easily cast in the framework of hazard maps by defining a hazard $V(\epsilon_s)$ associated with following more closely than this safe distance. When dealing with multiple lanes, this hazard must be interpolated to zero in some manner outside the lane to reflect the fact that the vehicle occupies only a single lane. Achieving this raises a new set of issues which are treated in Sections 3 and 4.

For now, we assume that the vehicle behaves like a mass point with one degree of freedom in the longitudinal direction and an equation of motion:

$$m\ddot{s} = u \tag{4}$$

The control input (physically the braking and acceleration forces) is assigned the negative gradient of the potential function or hazard, yielding the closed-loop dynamics

$$m\ddot{s} = -\frac{\partial V}{\partial \epsilon_s} = -\frac{\partial V}{\partial s} \tag{5}$$

To make this controller practical, several constraints on the potential function are useful. First, setting the gradient such that $\partial V/\partial \epsilon_s = 0$ when $\epsilon_s = 0$ ensures that the force applied to the vehicle as it hits the potential field is not discontinuous. Secondly, an increasing gradient with increasing spacing error leads to a gradual increase in brake force. Finally, the peak value of the gradient should be no greater than the maximum braking force of which the vehicle is capable. By adapting the shape of the potential function and choosing different gains in Equation 3, different following behaviors can be obtained.

Constant Time Headway

A constant time headway strategy can be obtained by setting $\lambda_1 = 0$ so

$$s_d = \lambda_2 \dot{s} + \lambda_3 \tag{6}$$

The rate of change of spacing error is given by:

$$\begin{aligned} \dot{\epsilon}_s &= \dot{s} - \dot{s}_l + \lambda_2 \ddot{s} \\ &= \dot{s} - \dot{s}_l - \frac{\lambda_2}{m}\frac{\partial V}{\partial \epsilon_s} \end{aligned} \tag{7}$$

The rate of change of kinetic and potential energy are

$$\dot{T} = m\dot{s}\ddot{s} = -\frac{\partial V}{\partial \epsilon_s}\dot{s} \tag{8}$$

$$\dot{V} = \frac{\partial V}{\partial \epsilon_s}\dot{s} - \frac{\partial V}{\partial \epsilon_s}\dot{s}_l - \frac{\lambda_2}{m}\left(\frac{\partial V}{\partial \epsilon_s}\right)^2 \tag{9}$$

The change of artificial potential energy or hazard contains three terms: the conservative transfer from kinetic energy and two nonconservative terms related to the motion of the potential field. The first of these nonconservative terms relates to the fact that the lead vehicle motion "pulls" the potential field along the roadway while the second relates to the motion of the potential field due to the time headway. Combining these equations, the total energy change is the sum of the nonconservative terms:

$$\dot{E} = -\frac{\partial V}{\partial \epsilon_s}\dot{s}_l - \frac{\lambda_2}{m}\left(\frac{\partial V}{\partial \epsilon_s}\right)^2 \tag{10}$$

If the lead vehicle is not moving in reverse, these terms clearly remove energy from the system, so the total hazard is bounded by the initial energy.

In fact, the maximum hazard will be less than the initial total energy since these damping terms are not negligible. A better bound on hazard can be obtained for a particular shape of the potential function. For instance, with a quadratic potential function

$$V(\epsilon_s) = \frac{1}{2} c_0 {\epsilon_s}^2 \tag{11}$$

the dynamics are given by

$$\dot{\epsilon}_s = \dot{s} - \dot{s}_l - \frac{\lambda_2}{m} c_0 \epsilon_s \tag{12}$$

$$\begin{aligned} \ddot{\epsilon}_s &= \ddot{s} - \ddot{s}_l - \frac{\lambda_2}{m} c_0 \dot{\epsilon}_s \\ &= -\frac{c_0}{m}\epsilon_s - \ddot{s}_l - \frac{\lambda_2}{m} c_0 \dot{\epsilon}_s \end{aligned} \tag{13}$$

so

$$\ddot{\epsilon}_s + \frac{\lambda_2 c_0}{m}\dot{\epsilon}_s + \frac{c_0}{m}\epsilon_s = -\ddot{s}_l \tag{14}$$

This can be solved for the maximum spacing error (and hence hazard) given any initial speed mismatch $\dot{\epsilon}_s$ and lead vehicle deceleration $\ddot{s}_l$. As Figure 1 illustrates, the maximum potential energy can be considerably less than the initial energy. This plot demonstrates the transfer from kinetic to artificial potential energy for a vehicle following a lead vehicle at 30 m/s with a 2 second headway after the lead vehicle begins to decelerate at 4 m/s^2. These simulations were performed using a simple vehicle model (Gerdes and Rossetter 1999) together with a modified Dugoff tire model (Guntur and Sankar 1980). The conservative nature of the energy bound follows from the motion of the lead vehicle and the time headway. In the event that both the time headway and the lead vehicle velocity are zero (corresponding to a constant desired safety distance and a "brick-wall" stop by the lead vehicle), the kinetic energy is transferred entirely into the hazard (Figure 2).

There are several points to note about this controller. First, while the potential field framework is generally associated with conservative forces, the construction of the artificial hazard around a safety distance that depends upon speed can inject damping into the system. Secondly, the shape of the potential function and the time headway are independent design parameters. Together they determine the absolute level of hazard experienced by the vehicle and the spacing error dynamics.

Conservative Safety Distance

A more conservative safety distance that can guarantee collision avoidance can be obtained with $\lambda_1 =$

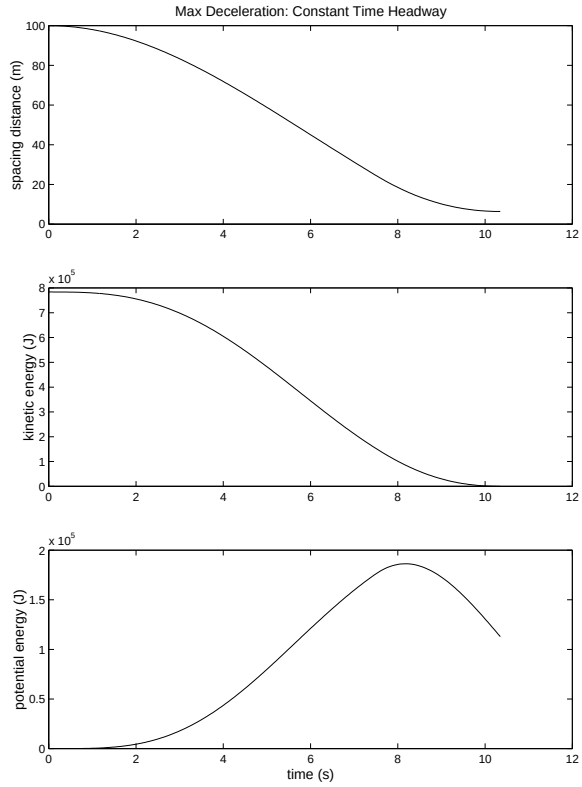

Figure 1: 4 m/s^2 deceleration at $30m/s$, $\lambda_2 = 2$

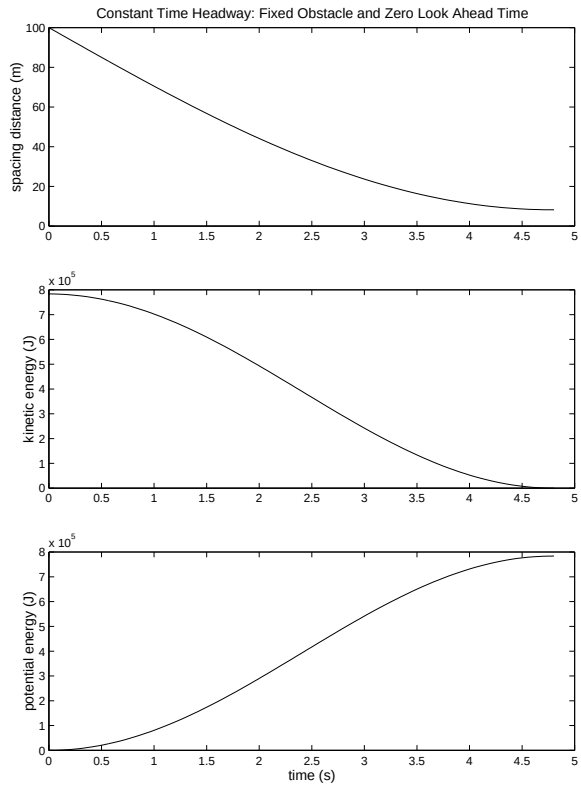

Figure 2: 'Brick wall' stop at $30m/s$, $\lambda_2 = 0$

$1/2d$ and $\lambda_2 = 0$:

$$s_d = \frac{1}{2d}\left(\dot{s}^2 - \dot{s}_l^2\right) + \lambda_3 \tag{15}$$

where d is the peak deceleration capability of both the leading and following vehicles. This strategy results in a spacing error given by

$$\dot{\epsilon}_s = \dot{s} - \dot{s}_l + \frac{1}{d}\left(\dot{s}\ddot{s} - \dot{s}_l\ddot{s}_l\right) \tag{16}$$

The rate of change of kinetic energy is the same as in the constant time headway case while the rate of change of potential energy is

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial \epsilon_s}\left(\dot{s} + \frac{1}{d}\dot{s}\ddot{s}\right) + \frac{\partial V}{\partial \epsilon_s}\left(-\dot{s}_l + \frac{1}{d}\dot{s}_l\ddot{s}_l\right) \\ &= \frac{\partial V}{\partial \epsilon_s}\dot{s} - \frac{1}{md}\left(\frac{\partial V}{\partial \epsilon_s}\right)^2 \dot{s} + \frac{\partial V}{\partial \epsilon_s}\left(\alpha - 1\right)\dot{s}_l \end{aligned} \tag{17}$$

where

$$\alpha = -\frac{\ddot{s}_l}{d} \leq 1 \tag{18}$$

This can also be bounded, though in a slightly different manner than the constant time headway case. From Equation 17, $\dot{V}$ is a maximum - not surprisingly - when the lead vehicle decelerates at its maximum level d. If the potential function has been designed to have a gradient equal to zero at zero spacing error and increasing thereafter as suggested, the rate of change of hazard goes to zero when

$$\frac{\partial V}{\partial \epsilon_s} = md \tag{19}$$

Figure 3 illustrates this follower law for the same set of conditions as Figure 1 with $d = 4m/s^2$. Since more conservatism has been introduced by this choice of the safety distance, the maximum hazard is even less than the constant time headway cases presented before.

A Note on Intelligent Cruise Control

The results presented here cover only half of an Intelligent Cruise Control system, namely the portion related to collision avoidance. The ability to maintain distance behind an accelerating vehicle must be added separately and necessarily requires an increase in the kinetic energy of the vehicle. If there is any overshoot associated with the follower law, this will also produce a transient increase in hazard. Thus while the collision avoidance part of an Intelligent Cruise Control can be viewed as a safety system in this energy theoretic framework, the ability to maintain distance behind an accelerating lead vehicle is best seen as a convenience feature.

In this context, the concept of hazard has a useful interpretation as a switch for convenience systems such as this. The system can simply be switched off when a given level of total energy (corresponding to the desired set speed or a combination of speed and hazard) is reached. Thus an Intelligent Cruise Control can be added to this framework and shut off when it begins to increase the hazard bound or kinetic energy beyond acceptable levels.

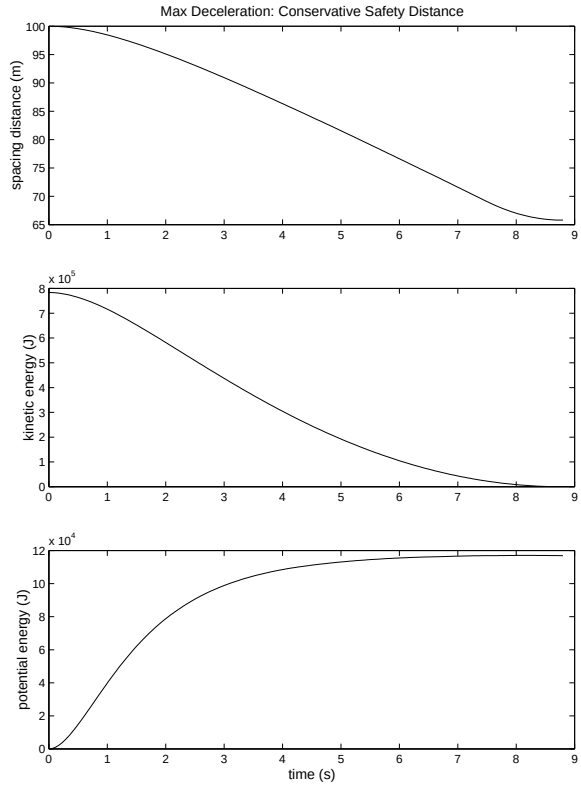

Figure 3: 4 m/s^2 deceleration at $30m/s$, $d = 4m/s^2$

3 Moving Hazards in 2D

Incorporating the hazard interpretation of vehicle following in a framework that includes lanekeeping and lane changing requires a couple of modifications. First, the model and control law must be extended to include yaw and lateral dynamics as briefly developed below (and in more detail in Gerdes and Rossetter (1999)). Secondly, the field associated with the lead vehicle must be truncated outside the lane to enable overtaking maneuvers. This is accomplished by a multiplicative shaping function on the follower potential function.

Vehicle and Environment Models

The vehicle model (Figure 4) is a simple yaw plane representation with three degrees of freedom (Koepele and Starkey 1990) and differential braking capability. While the double-track model is used for the purposes

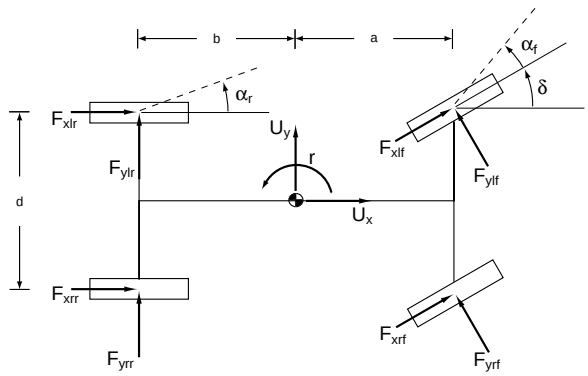

Figure 4: Planar Model of Vehicle Dynamics

of differential braking, we assume that the left and right tires possess the same slip angle (in other words, a simple bicycle model for the kinematics). Assuming a vehicle with throttle-, brake- and steer-by-wire capability, the equations of motion are

$$D\ddot{q} = f(\dot{q}) + g(\dot{q}, u_c) \tag{20}$$

where $\dot{q} = [U_x \; U_y \; r]^T$, the positive definite mass matrix, D, is

$$D = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \tag{21}$$

the drift vector f is:

$$f = \begin{bmatrix} mrU_y \\ F_{yr} - mrU_x + \hat{F}_{yf} \\ -bF_{yr} + a\hat{F}_{yf} \end{bmatrix} \tag{22}$$

where

$$\hat{F}_{yf} = -\hat{C}_f \left(\frac{ra + U_y}{U_x} \right) sgn(U_x) \tag{23}$$

for some effective cornering stiffness, $\hat{C}_f \geq 0$ and g can be set arbitrarily by control. Using this definition, the drift term f represents system damping since

$$\begin{aligned} \dot{q}^T f &= F_{yr}(U_y - br) + \hat{F}_{yf}(U_y + ra) \\ &= F_{yr}(U_y - br) - \hat{C}_f \frac{(U_y + ra)^2}{|U_x|} \\ &\leq 0 \end{aligned}$$

under the assumption that the tire force and the slip angle are oppositely directed.

The environment is modeled simply as a straight section of roadway with the position vector $w = [s \; e \; \psi]^T$ representing the location of the vehicle cg in the environment. The state vector of the system is therefore given in terms of the position variables, w and the velocity vector $\dot{q}$. Transformation between the environmental and body fixed systems can be achieved through

$$\frac{\partial \dot{w}}{\partial \dot{q}} = \frac{\partial w}{\partial q} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{24}$$

Control Law

The controller form proposed by Gerdes and Rossetter (1999) adds a conservative force derivable from the potential function and a general damping term to the existing vehicle dynamics. The general control law is of the form

$$g(\dot{q}, u_c) = B_d(\dot{q}, u_d) + F(w, \dot{q}, u_d) - \left(\frac{\partial V}{\partial w} \frac{\partial w}{\partial q} \right)^T \tag{25}$$

where $V(w, t)$ is the potential function describing the overall hazard in the environment and $F(w, \dot{q}, u_d)$ is a generalized damping term. This term can be any vector function that satisfies

$$\dot{q}^T F(w, \dot{q}, u_d) \leq 0 \tag{26}$$

Here it is simply set to zero. Since we have assumed throttle-, brake- and steer-by-wire, the driver commands for steering, acceleration and braking must also be included in the control law and these appear through B_d.

It can be shown that if the following relation holds for the follower law:

$$\frac{\partial V}{\partial \epsilon_s} (-\dot{s}_l + \dot{s}_d) \leq 0 \tag{27}$$

the combination of kinetic energy and artificial potential energy is always decreasing in the absence of driver input. Hence the results of Gerdes and Rossetter (1999) can be simply extended. As discussed below, however, this condition is difficult to strictly enforce (though generally true) and the bound is not necessarily useful for design.

4 Simulation

Potential Field Control

The particular potential function chosen to illustrate the combination of fixed and moving hazards in two dimensions has the form:

$$V = V_e(e) + V_\epsilon(\epsilon_s) V_l(e) \tag{28}$$

where V_e is the lanekeeping potential field, V_ϵ is the vehicle following field and V_l shapes the vehicle field to match the lane. The exact function used is in Figure 5 and represents a combination of the 2 second time headway controller for a vehicle in the right lane with lanekeeping fields for left and right lanes as used in Gerdes

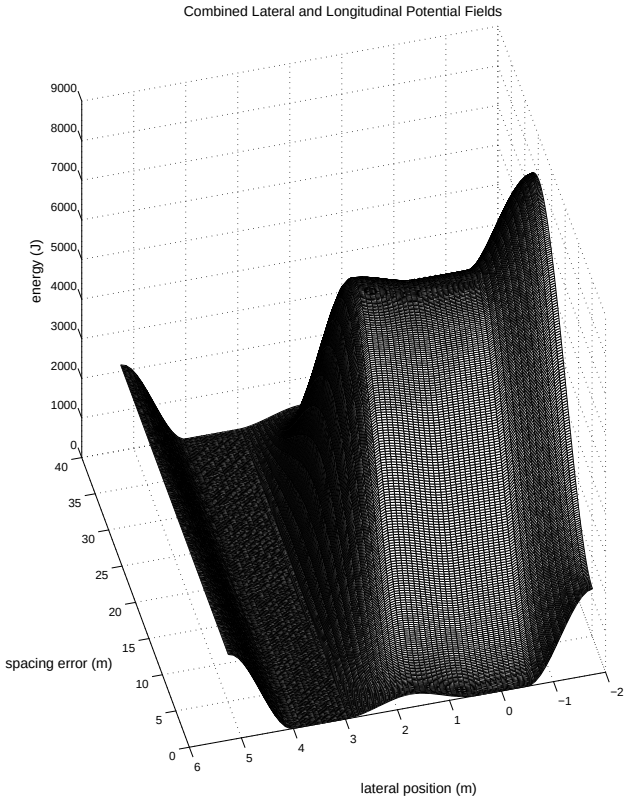

Figure 5: Section of Lanekeeping Potential Function

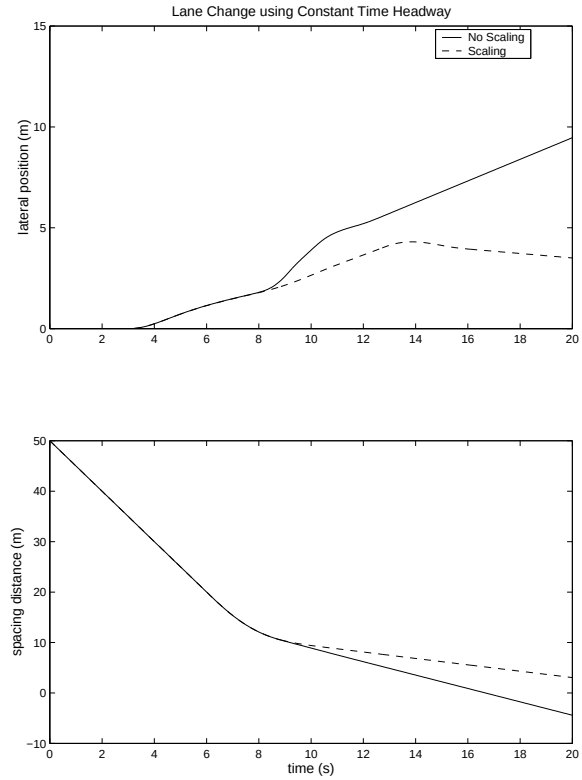

Figure 6: Driver Lane Change with Combined Fields

and Rossetter (1999). With this form of potential function, the equations of motion of the vehicle in global coordinates become:

$$m\ddot{s} = -V_l \frac{\partial V_\epsilon}{\partial \epsilon_s} - \left(F_{yr} + \hat{F}_{yf}\right) \sin\psi \tag{29}$$

$$I_z \ddot{\psi} = -\left(-bF_{yr} + a\hat{F}_{yf}\right) \cos\psi \tag{30}$$

$$m\ddot{e} = -\left(\frac{\partial V_e}{\partial e} + V_\epsilon \frac{\partial V_l}{\partial e}\right) + \left(F_{yr} + \hat{F}_{yf}\right) \cos\psi \tag{31}$$

A closer examination of Figure 5 reveals a major obstacle to merging lanekeeping and vehicle following. As the vehicle moves closer to the lead vehicle, the hazard increases sharply. If the driver changes lanes after entering the vehicle following field, the potential field controller translates this artificial potential energy into kinetic energy by greatly increasing the lateral velocity. The effects of this can be seen in the solid lines of Figure 6 which shows a driver-induced lane change at 3 sec, slightly before the following vehicle is influenced by the potential field. The controller moves the car out of the right hand lane, but overshoots the left hand lane, plowing through the potential field on the opposite side.

This difficulty arises due to the inherent asymmetry of vehicle dynamics. Most of the kinetic energy is associated with the longitudinal momentum, so the potential energy grows fairly large during braking. In contrast, only a small distance is necessary to change lanes, resulting in a very high gradient in the lateral direction and a large increase in lateral momentum. This behavior, while necessary to ensure that the total energy decreases, is not particularly acceptable or helpful as a driver assistance system. There is no particular reason why the rapid reduction in hazard needs to be applied as a lateral force as opposed to simply being dissipated. Similarly, the steep gradient in the lateral direction produces far more force than is necessary to prevent the vehicle from a sideswipe collision with the vehicle as it continues the overtaking maneuver. Since most of the energy is associated with longitudinal motion, a lower gradient is sufficient to keep the car in its own lane. These issues are analagous to the points raised by Schiller *et al.* (1998*b*) with regards to the difficulty in tuning virtual bumper springs so that collision avoidance did not force the vehicle out of the lane. A similar effect would be seen for this controller if the lanekeeping potential fields were applied to curved roads as opposed to the straight segments under consideration here.

Scaled Potential Field Control

This problem can be resolved by scaling down the gradient of the follower potential field in the lateral di-

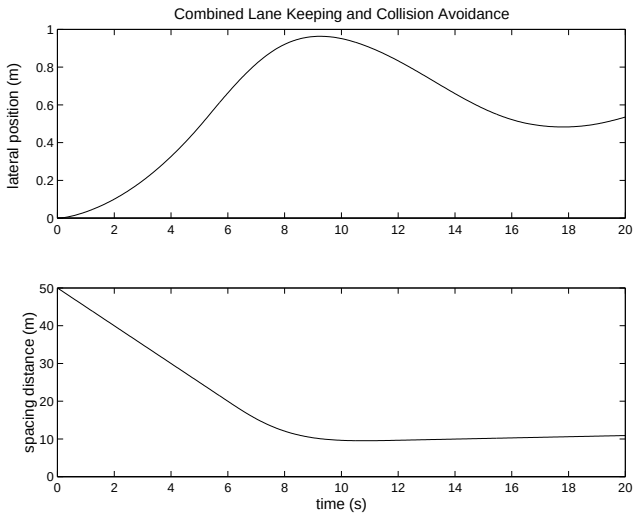

Figure 7: Lanekeeping During Approach

rection in the control law. This changes Equation 31 to

$$m\ddot{e} = -\left(\frac{\partial V_e}{\partial e} + c_\epsilon V_\epsilon \frac{\partial V_l}{\partial e}\right) + \left(F_{yr} + \hat{F}_{yf}\right)\cos\psi \quad (32)$$

by adding a constant $c_\epsilon \leq 1$ while leaving Equations 29 and 30 unchanged. As can be seen from the dotted line in Figure 6, this does not result in a dramatic increase in lateral velocity when changing lanes and the vehicle stays in the potential fields in the left hand lane. Furthermore, this scaling produces a system that can easily keep the car in the lane while approaching a vehicle ahead, as demonstrated in Figure 7.

It might appear that this solution throws out the very concept of unified hazard that motivated this work. With environmental forces scaled in only one direction, the overall decrease of energy (kinetic plus artificial potential) is no longer guaranteed. Clearly, if a vehicle moves laterally into the field around the other vehicle in Figure 5, it will experience an increase in hazard greater than the decrease in kinetic energy. This increase, however, is *bounded*, and bounded far below the level of initial kinetic energy that would be predicted by an overall energy bound.

To see why this is the case, consider the form of Equations 29 and 32. When the potential functions are chosen to have increasing slopes below their saturation values (possessing a quadratic form, for instance), the gradient terms take the form of springs. Thus except for a few small terms related to the tire forces that are not cancelled (and can be meaningfully bounded), the longitudinal dynamics are decoupled from the lateral and yaw dynamics. The lateral and yaw dynamics are themselves decoupled for a neutral steering car and can be stabilized and approximately decoupled by applying the control forces at the neutral steer point (Rossetter and Gerdes 2000) even if the vehicle oversteers by nature.

Thus we can design combined controllers by setting lanekeeping fields and the lateral and longitudinal components of moving obstacle fields such that the level of hazard produced by specified operating conditions lies beneath an acceptable bound. This is in some sense similar to tuning the springs and dampers of a virtual bumper design, though these springs are attached to global coordinates, move relative to a desired safety distance and can be related to a uniform notion of hazard. This uniform notion of hazard also gives a specific numerical objective to the design procedure. While design can be performed through simulation, the form of the equations of motion enables a more analytical treatment by considering the energy associated with a particular coordinate as a Lyapunov function. In this framework, the coupling effects can be treated as disturbances and more rigorous bounds determined. This is a subject of current research.

5 Conclusions / Future Work

By incorporating a safety distance with a potential function, the idea of environmental hazards suggested by Reichardt and Schick (1994) and developed into assistance systems by Gerdes and Rossetter (1999) can be extended to include moving vehicles. In this formulation, a variety of existing follower laws can be incorporated into the framework of a hazard map. We believe that such a formulation offers several advantages:

1. The hazard map provides a common measure for examining and weighting vehicle control actions.
2. A threshold combination of kinetic energy and hazard can be used as a signal to turn off features such as Intelligent Cruise Control that can add energy or hazard to the system.
3. When extending system integration into regions of tire saturation, the hazard value provides useful information on how to saturate by weighting the relative importance of lateral position, longitudinal position and yaw.

The asymmetry problem in creating fields that combine lateral and longitudinal effects leads us to look at designing these components separately using a Lyapunov approach with bounds on the coupling. This will incorporate the energy in different modes of the system and ensure safe and acceptable energy transfer.

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