The Role of Handling Characteristics in Driver Assistance Systems with Environmental Interaction

Eric J. Rossetter

Design Division
Department of Mechanical Engineering
Stanford University
Stanford, California 94305-4021
Email: ejross@cdr.stanford.edu

J. Christian Gerdes

Design Division
Department of Mechanical Engineering
Stanford University
Stanford, California 94305-4021
Email: gerdes@cdr.stanford.edu

Abstract

Future automotive safety functions, such as lanekeeping or collision avoidance, link the vehicle dynamically to its environment. As a result, vehicle motion is determined by a combination of the mechanical system dynamics and the virtual link to the roadway or obstacles. The need for careful design of both the mechanical and control system elements is particularly apparent for these driver assistance systems since control is intended to complement, not cancel, the vehicle dynamics. This paper presents stability requirements for vehicles with a driver assistance system that keeps the vehicle in the lane through the application of virtual forces. When the application point is at the center of gravity, the results indicate a critical speed for understeering vehicles and instability for oversteering vehicles. Shifting the application point of the virtual force to the neutral steer point, however, stabilizes the oversteering case under a critical speed.

1 Introduction

Today, many vehicles are being equipped with driver assistance systems such as ABS and stability control. These systems are designed to aid the driver by preventing any unstable or unpredictable vehicle behavior. Although they provide stable dynamics, these control systems do not prevent the vehicle from avoiding hazardous environmental obstacles. Gerdes and Rossetter [1] proposed a method for incorporating such links to the environment within the paradigm of artificial potential fields. In this work, the fields represent virtual forces from the environment. This is closely related to previous work in robotics (e.g. Khatib [5] and Hogan [4]) with the objective being a nominally safe driving environment as opposed to end-effector placement or trajectory generation. Similar ideas have been proposed by Reichardt and Schick [6] for autonomous vehicles and by Hennessey et al [3] for the design of a 'virtual bumper'.

Since the driver still has high level control in such a system, the combined dynamics resulting from physical and virtual forces are of primary importance. These dynamics not only determine the stability from the driver's perspective but also specify the response to environmental obstacles. Within the potential field framework, the hazard associated with objects in the environment can be interpreted as an artificial potential function and a controller designed to ensure that an artificial energy (consisting of the actual kinetic energy and the artificial potential) is bounded [1]. Hence, the hazard experienced by the vehicle can also be bounded. For the case of lanekeeping, the conservatism in this bound can be directly traced to the handling dynamics of the vehicle. An understeering car will yaw out of the field as it returns to the center of the lane and hence not transfer much of the kinetic energy into potential energy (or hazard). Conversely, an oversteering car will yaw into the field and transfer all of the vehicle's kinetic energy into artificial potential energy. From the driver's perspective, the oversteering vehicle with virtual forces for lanekeeping is unstable.

This paper presents a stability analysis of the system to demonstrate the handling implications of virtual forces for lanekeeping. The results clearly illustrate the instability associated with an oversteering vehicle and the presence of a critical speed for an understeering vehicle. Shifting the application point of the virtual forces to the neutral steering point of the vehicle, however, restores stability to the oversteering car below a new critical speed. These results provide insight into the proper way of applying virtual forces to control high speed vehicles and the connections between such systems and the vehicle handling characteristics. While discussed in terms of potential fields, the same concepts apply to other two-dimensionsal collision avoidance systems such as the 'virtual bumper' approach used by Hennessey et al [3] when the driver remains in the loop.

2 Vehicle Dynamics

The vehicle model used in the analysis is a simple three degree of freedom yaw plane representation with differential braking shown in Figure 1.

$$m\dot{U}_x = F_{xr} + F_{xf} cos\delta - F_{yf} sin\delta + mrU_y \quad (1)$$

$$m\dot{U}_y = F_{yr} + F_{xf} sin\delta + F_{yf} cos\delta - mrU_x \quad (2)$$

$$I_z\dot{r} = aF_{xf}\sin\delta + aF_{yf}\cos\delta - bF_{yr} \quad (3)$$
$$+ \frac{d}{2}(\Delta F_{xr} + \Delta F_{xf}\cos\delta)$$

where

$$F_{xf} = F_{xrf} + F_{xlf} \quad (4)$$

$$F_{xr} = F_{xrr} + F_{xlr} \quad (5)$$

$$\Delta F_{xf} = F_{xrf} - F_{xlf} \quad (6)$$

$$\Delta F_{xr} = F_{xrr} - F_{xlr} \quad (7)$$

The parameters are illustrated in Figure 1.

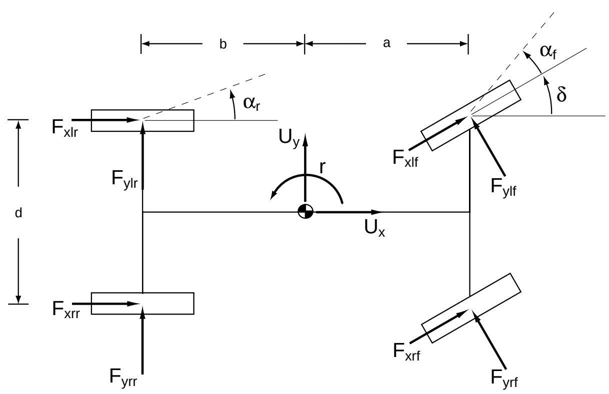

Figure 1: Vehicle Model

Assuming small angles and equal slip angles on the left and right wheels,

$$\alpha_f = \frac{U_y + ra}{U_x} - \delta \quad (8)$$

$$\alpha_r = \frac{U_y - rb}{U_x} \quad (9)$$

Using a linear tire model and assuming the same cornering stiffness, C_y, on both tires, the lateral forces are given as

$$F_{yf} = -C_y\alpha_f \quad (10)$$

$$F_{yr} = -C_y\alpha_r \quad (11)$$

Substituting the expressions for the lateral forces into Equations 1 through 3 and ignoring higher orders of δ yields,

$$m\dot{U}_x = mrU_y + F_{xr} + F_{xf} + C_y(\frac{U_y + ra}{U_x})\delta \quad (12)$$

$$m\dot{U}_y = -C_y(\frac{U_y - rb}{U_x}) - C_y(\frac{U_y + ra}{U_x}) - mrU_x \quad (13)$$
$$+ C_y\delta + F_{xf}\delta$$

$$I_z\dot{r} = aF_{xf}\delta - aC_y(\frac{U_y + ra}{U_x}) + bC_y(\frac{U_y - rb}{U_x}) \quad (14)$$
$$+ aC_y\delta + \frac{d}{2}(\Delta F_{xr} + \Delta F_{xf})$$

Assuming a vehicle that has throttle, brake, and steer-by-wire capabilities, the equations can be rewritten as

$$D\ddot{q} = f(\dot{q}) + g(\dot{q}, u_c) \quad (15)$$

where $\dot{q} = [U_x \; U_y \; r]^T$ and the control vector $u_c = [\delta \, F_{xrf} \, F_{xlf} \, F_{xrr} \, F_{xlr}]^T$. D is the positive definite mass matrix, $f(\dot{q})$ contains the terms that are not influenced by the control vector and $g(\dot{q}, u_c)$ has the remaining controlled terms.

$$D = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

$$f(\dot{q}) = \begin{bmatrix} mrU_y \\ -C_y(\frac{U_y - rb}{U_x}) - C_y(\frac{U_y + ra}{U_x}) - mrU_x \\ -aC_y(\frac{U_y + ra}{U_x}) + bC_y(\frac{U_y - rb}{U_x}) \end{bmatrix}$$

$$g(\dot{q}, u_c) = \begin{bmatrix} F_{xr} + F_{xf} + C_y(\frac{U_y + ra}{U_x})\delta \\ C_y\delta + F_{xf}\delta \\ aF_{xf}\delta + aC_y\delta + \frac{d}{2}(\Delta F_{xr} + \Delta F_{xf}) \end{bmatrix}$$

3 Control Law

The potential function used in the control law is described in global coordinates $w = [s \; e \; \psi]^T$ where s is the distance along the roadway, e is the distance of the vehicle's center of gravity from the lane center and ψ is the heading angle. Transformation between the global and body fixed coordinates is achieved with

$$\frac{\partial \dot{w}}{\partial \dot{q}} = \frac{\partial w}{\partial q} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

Assuming that the vehicle is not being influenced by the driver and, assuming no damping is added to the system, the control is simply the conservative forces derived from the potential function:

$$g(\dot{q}, u_c) = -\left(\frac{\partial V}{\partial w}\frac{\partial w}{\partial q}\right)^T \quad (17)$$

where V(w) is the potential function representing hazard in the global reference frame. With this control law, the vehicle dynamics can be rewritten, replacing the control input vector with the potential field:

$$D\ddot{q} = f(\dot{q}) - \left(\frac{\partial V}{\partial w}\frac{\partial w}{\partial q}\right)^T \quad (18)$$

This set of non-linear differential equations is the focus of the analysis. The behavior in question is the stability of the vehicle with respect to a desired trajectory consisting of a constant longitudinal velocity at the minimum of the potential function (in other words, staying in the center of the lane on a straight section of road). Since the interesting behavior is in the global coordinates, it makes sense to transform the equations of motion into global states and then linearize the system about the desired trajectory.

4 Transformation and Linearization of the Dynamics

The basic idea is to transform the equations of motion into the global reference frame and then rewrite the three second order differential equations as six first order equations that are functions of $[e\ \dot{e}\ s\ \dot{s}\ \psi\ \dot{\psi}]$. Jacobian linearization can then be performed about a straight trajectory where $\dot{s}$ equals a constant S and all other states are zero. This trajectory will correspond to the minimum of a potential function. In the absence of a potential function, this analysis returns well-known stability results for vehicle handling, as should be expected. The system will then be analyzed using a second order potential function that is only dependent on the lateral position.

$$V(e) = Ke^2 \tag{19}$$

where K is a positive constant. This simple quadratic potential function gives a linear conservative force which does not disappear in the system linearization.

A technique will then be discussed to transform the application of the potential field to the neutral steer point. This will achieve the desired behavior in an oversteering vehicle.

4.1 Linearization Without Potential Fields

The linearization of the system about a constant longitudinal velocity gives,

$$\delta\dot{x} = A\delta x \tag{20}$$

where $\delta x = [\delta e\ \delta\dot{e}\ \delta\psi\ \delta\dot{\psi}]^T$ and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-2C_y}{mS} & \frac{2C_y}{m} & \frac{(-a+b)C_y}{mS} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{(-a+b)C_y}{I_zS} & \frac{(a-b)C_y}{I_z} & \frac{-(a^2+b^2)C_y}{I_zS} \end{bmatrix} \tag{21}$$

Taking the determinant of $(\lambda I - A)$ yields the characteristic equation of the system.

$$\lambda^2\left(\lambda^2 + \lambda a_1 + a_2\right) = 0 \tag{22}$$

where,

$$a_1 = \frac{2C_yI_z + (a^2+b^2)C_ym}{I_zmS}$$
$$a_2 = \frac{{C_y}^2(a+b)^2 + (b-a)C_ymS^2}{I_zmS^2}$$

From the above equation it can be determined that there are two eigenvalues at 0 due to the two positional states. For an understeering car, where $b > a$, both coefficients a_1 and a_2 are positive which, for a second order system, is sufficient to prove stability. In an oversteering case ($a > b$), the coefficient a_2 will be negative when

$$S > \sqrt{\frac{C_y(a+b)^2}{(a-b)m}} \tag{23}$$

This is the well known critical speed for an oversteering vehicle (Gillespie [2]). As expected, the transformation to global coordinates did not change the basic stability properties of the vehicle. Once the vehicle is linked to the environment through the potential field, however, the eigenvalues corresponding to handling and positional states are no longer independent.

4.2 Linearization With a Potential Function

Adding a potential function of the form described in Equation 19 yields a linear system which has one extra term appearing in the second row of the matrix given in Equation 21. Since the potential field is oriented in the lateral direction it only enters the second differential equation.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-2K}{m} & \frac{-2C_y}{mS} & \frac{2C_y}{m} & \frac{(-a+b)C_y}{mS} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{(-a+b)C_y}{I_zS} & \frac{(a-b)C_y}{I_z} & \frac{-(a^2+b^2)C_y}{I_zS} \end{bmatrix} \tag{24}$$

The characteristic equation of this matrix is

$$\lambda^4 + \lambda^3 b_1 + \lambda^2 b_2 + \lambda b_3 + b_4 = 0 \tag{25}$$

where

$$b_1 = \frac{2C_yI_z + (a^2+b^2)C_ym}{I_zmS}$$
$$b_2 = \frac{{C_y}^2(a+b)^2 + (b-a)C_ymS^2 + 2KI_zS^2}{I_zmS^2}$$
$$b_3 = \frac{2KC_y(a^2+b^2)}{I_zmS}$$
$$b_4 = \frac{2KC_y(b-a)}{I_zm}$$

Regardless of the vehicle's speed, the last term, b_4, is always negative when oversteering parameters are used. Sign changes in the characteristic equation coefficients fail the necessary conditions for stability. The addition of the potential field thus alters the oversteering vehicle dynamics to produce instability by lowering the critical speed to zero.

If the vehicle is understeering, all the coefficients are positive. Although this is a necessary condition for stability, it is not sufficient. Using the Routh array, it can be shown that the system is stable up to a critical velocity.

$$S_{cr} = \frac{1}{\sqrt{2}}\left(\frac{N}{D}\right)^{1/2} \tag{26}$$

where N and D are

$$N = -(a+b)^2(a^2+b^2)C_y^2(2I_z + (a^2+b^2)m)$$
$$D = I_z(2I_z((a-b)C_y + (a^2+b^2)K)$$
$$+ \ (a-b)(a^2+b^2)C_ym)$$

The fact that even understeering cars have conditions for stability is extremely important in the design of these types of controllers for high speed applications. For the parameters used in the simulation (Table 1), the critical speed is

$47.47m/s$, which is fairly large. Adding damping to the control vector will help stabilize the system when it is past the critical speed predicted here.

The unstable behavior in the oversteering vehicle occurs because the potential field is applying a virtual control force at the center of gravity, using a combination of steering and differential braking inputs. In order to rectify this problem, the virtual force can be shifted to a point other than the center of gravity. Moving the application point of the potential field to the neutral steer point gives the desired response for an oversteering car and does not violate the proof of nominal safety given by Gerdes and Rossetter [1].

4.3 Shifting the Potential Field Application

In order to shift the application of the potential field, there needs to be a transformation from the new application point in global coordinates to the body fixed coordinates. The new application point on the vehicle will be given as $\tilde{w} = [\tilde{s}\ \tilde{e}\ \tilde{\psi}]^T$. Introducing a transformation matrix from the global velocity coordinates of the center of gravity to the global velocity coordinate of the new point as

$$\frac{\partial \tilde{w}}{\partial w} = \begin{bmatrix} 1 & 0 & -x_{ns} sin\psi \\ 0 & 1 & x_{ns} cos\psi \\ 0 & 0 & 1 \end{bmatrix} \tag{27}$$

we can transform the potential field from the new application point to body fixed velocity coordinates so that

$$\dot{V}(\tilde{w}) = \left(\frac{\partial V}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} \frac{\partial w}{\partial q} \right) \dot{q} \tag{28}$$

Assuming that the potential function is differentiable, the system still exhibits nominally safe behavior. If the potential function $V(\tilde{w})$ is interpreted as a level of hazard applied to system states, then in the absence of driver input, the system hazard is bounded by:

$$V_{max} = \frac{1}{2} \dot{q}(0)^T D \dot{q}(0) + V(\tilde{w}(0))$$

where $\tilde{w}(0)$ and $\dot{q}(0)$ are the values at the initial time $t = 0$.

Proof Defining an effective energy by

$$E = \frac{1}{2} \dot{q}^T D \dot{q} + V(\tilde{w})$$

the rate of change of energy is:

$$\begin{aligned} \dot{E} &= \dot{q}^T D \ddot{q} + \frac{\partial V}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} \frac{\partial w}{\partial q} \dot{q} \\ &= \dot{q}^T \left[f(\dot{q}) + g(\dot{q}, u_c) \right] + \frac{\partial V}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} \frac{\partial w}{\partial q} \dot{q} \\ &= \dot{q}^T \left[f(\dot{q}) - \left(\frac{\partial V}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} \frac{\partial w}{\partial q} \right)^T \right] + \frac{\partial V}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} \frac{\partial w}{\partial q} \dot{q} \\ &= \dot{q}^T f(\dot{q}) \\ &\leq 0 \end{aligned}$$

In Gerdes and Rossetter [1], it is shown that $\dot{q}^T f(\dot{q})$ is always negative. Since the energy cannot increase,

$$V_{max} \leq E_0 = \frac{1}{2} \dot{q}(0)^T D \dot{q}(0) + V(\tilde{w}(0))$$

bounds the hazard in the system.

4.4 Linear Analysis of the Modified System

The neutral steer point of a vehicle is the point on the centerline where an external force will create no steady state yaw velocity. This concept is often used to discuss sidewind sensitivity of a vehicle and has a natural interpretation when considering virtual forces and stability. With the potential field acting at the neutral steer point, x_{ns}, where

$$x_{ns} = \frac{a-b}{2} \tag{29}$$

the equations of motion have extra components appearing in the moment equation. The transformation and linearization can be performed exactly as before. Now the matrix given in Equation 24 contains an extra component in the last row due to the moment created by the potential field:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-2K}{m} & \frac{-2C_y}{mS} & \frac{2C_y}{m} & \frac{(-a+b)C_y}{mS} \\ 0 & 0 & 0 & 1 \\ \frac{-(a-b)K}{I_z} & \frac{(-a+b)C_y}{I_z S} & \frac{(a-b)C_y}{I_z} & \frac{-(a^2+b^2)C_y}{I_z S} \end{bmatrix} \tag{30}$$

The characteristic equation is

$$\lambda^4 + \lambda^3 c_1 + \lambda^2 c_2 + \lambda c_3 + c_4 = 0 \tag{31}$$

where

$$\begin{aligned} c_1 &= \frac{2C_y I_z + (a^2+b^2) C_y m}{I_z m S} \\ c_2 &= \frac{{C_y}^2 (a+b)^2 + (b-a) C_y m S^2 + 2K I_z S^2}{I_z m S^2} \\ c_3 &= \frac{K C_y (a+b)^2}{I_z m S} \\ c_4 &= 0 \end{aligned}$$

The only term in this system which can possibly go negative for an oversteering vehicle is c_2. From looking at the Routh array, the system actually has a critical speed that occurs before c_2 becomes negative.

$$S_{cr} = \left(\frac{N}{D} \right)^{1/2} \tag{32}$$

where

$$\begin{aligned} N &= -(a+b)^2 C_y^2 (2I_z + (a^2+b^2) m) \\ D &= 4 I_z^2 K + (a-b) I_z (-2C_y + (a-b) K) m \\ &- (a-b)(a^2+b^2) C_y m^2 \end{aligned}$$

Using the parameters in Table 1, the critical speed of the vehicle is $31.94m/s$. Below this speed, the system is stable.

	Understeer	Oversteer
m (kg)	1670	1670
I_z (N/m^2)	2100	2100
C_y (N/rad)	61595	61595
K	5000	5000
a (m)	1.3	1.7
b (m)	1.7	1.3

Table 1: Vehicle Parameters

As the application of the force is shifted forward on the oversteering vehicle, the positive eigenvalues that appeared when the force was applied at the center of gravity are shifted to the left on the real axis until they go negative. It can be shown that the oversteering vehicle in the potential field will not be stabilized until the application of the force is at the neutral steer point of the vehicle.

5 Simulation

In order to see the responses of the vehicle, simulations were run with the quadratic potential function used in the analysis (Equation 19). Three different conditions are shown in Figure 2. The first two show the responses for an understeering and oversteering vehicle if the potential field is applied at the center of gravity. The third case shows the oversteering vehicle with the potential field applied at the neutral steer point. The initial conditions for the simulation are $e = 0.5m$ and $S = 20m/s$, representing a normal highway speed and an initial offset from the lane center. All other states are zero. The parameters used in the simulation are in Table 1.

The results shown in Figure 2 show the exact behavior predicted by the linear analysis on the system. When the potential field is applied at the center of gravity, the understeering vehicle is stable and tends to oscillate about the minimum of the potential function. Use of an oversteering vehicle exhibits drastically different results. The force from the potential field initially pushes the car towards the minimum of the function. The change in the vehicle dynamics, however, causes a rotation into the potential function. The large amount of energy that exists from the longitudinal speed is then transfered into the lateral direction and the vehicle moves further into the field. Assuming a potential function that exists for all e, the oversteering vehicle would turn into the potential field until the longitudinal axis of the vehicle is parallel to the gradient of the potential field. In essence, the oversteering vehicle will transfer all of its initial kinetic energy into artificial potential energy.

Transferring the application of the potential field to the neutral steer point creates a stable, well behaved response. In fact, the response of the oversteering vehicle is now more desirable than the understeering case. This results because the application of the potential field at the neutral steer point

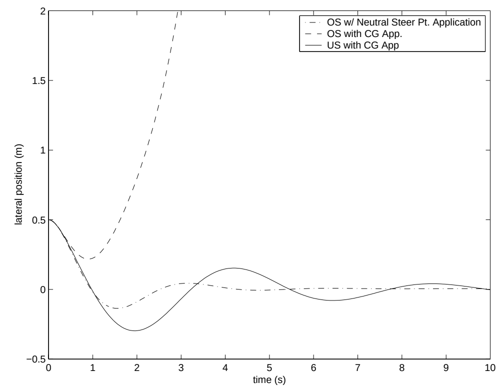

Figure 2: Vehicle Model

creates a zero yaw velocity at steady state. Reducing the amount of rotation that occurs lowers the energy transfer from the vehicle into the potential field.

6 Concluding Remarks

The analysis in this paper illustrates the instabilities that can occur when virtual control forces are applied to a vehicle. The proposed solution - altering the point of application of the potential field or virtual force - has many uses in this type of control. Not only does it stabilize the oversteering vehicle, it also alters the transient behavior of any vehicle within the potential field. Future work will attempt to bound the energy transfer in the system based on the application point of the field. A possible next step is to dynamically alter the application point of the potential field based on the behavior of a vehicle with unknown parameters in order to provide both lanekeeping and yaw control.

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