IMPROVING REALITY-BASED MODELS FOR VIBRATION FEEDBACK

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ABSTRACT

Reality-based modeling of vibrations can be used to enhance the haptic display of hard surfaces in virtual environments. In this work, we propose modifying reality-based vibrations through psychophysical experiments. Virtual vibration models were developed from empirical vibration waveforms measured by tapping on real surfaces with the instrumented stylus of a three degree-of-freedom haptic display. The haptic display was then evaluated on its ability to display the desired waveforms. Psychophysical experiments utilizing the display were performed to evaluate the realism of various vibration parameter values used in the vibration model. From these experiments, it was discovered that the best parameters differed from those of the original reality-based model. Additional experiments verified the effectiveness of vibration feedback by showing that subjects could differentiate between materials in a virtual environment.

1 INTRODUCTION

1.1 Motivation

Conventional approaches to haptic display usually consist of designing a virtual model with simple geometry and analytical equations, then using a stiffness control law for making hard surfaces. However, such environments often lack the realism of actual conditions. With common haptic interfaces, surfaces often feel “squishy” or unrealistically smooth. One solution to improving the realism of such environments is to use surface models based on physical measurements. Empirical data can be used to model textures, surface geometry, and the vibrations resulting from performing various tasks. This work discusses the design of physically-based vibration feedback algorithms for making surfaces appear hard.

Hard surfaces are difficult to display with conventional non-passive haptic (or force-feedback) devices. Due to limited actuator power, system delays, and possible instabilities, the display of hard surfaces is always limited in stiffness, and therefore, in realism. The addition of vibration feedback can allow haptic devices to emulate the feeling of real surfaces without resorting to hardware modifications.

When humans touch an environment, fast-acting sensors embedded in the skin record the minute vibrations occurring from this interaction. When tapping on a table top, the presence of the table is detected not only by kinesthetic sensors in the muscles and tactile sense due to skin deformation, but also from vibrations resulting from the contact of two surfaces (the finger and the table) that can be both heard and felt. Haptic displays are often good at providing kinesthetic sensations, but the amount of force is limited in comparison to the elastic forces provided by many real materials. In addition, most current haptic displays do not include tactile feedback in the form of skin deformation. Vibration feedback, however, is an enhancement to virtual environments that can be accomplished without additional hardware for most haptic displays.

Several questions are addressed in this research:

- What do impact vibrations look like, and how should they be modeled?
- How good does a haptic display have to be in order to display vibration feedback models?
- How can vibration feedback be made to feel as realistic as possible?
- Is vibration feedback effective?

In this paper, we first present the measurement and modeling of impact vibrations for a select set of materials. Next, we discuss the requirements of a haptic device for displaying vibrations and provide a characterization of the experimental apparatus. For models which cannot be displayed with the haptic device, the parameters are scaled. We then present psychophysical experiments used to evaluate and select vibration parameter values that feel most realistic for each material. Experiments were performed to verify the effectiveness of vibration feedback by showing that subjects could differentiate between materials in a virtual environment. We conclude with a discussion of some of the fundamental issues associated with vibration feedback.

1.2 Related Work

In recent years there have been several studies of vibration feedback in both the acoustic and tactile domains. Durst and Krotkov[4] studied the impact acoustics resulting from object collisions and were able to develop an object classification system from the data. For the application of haptic display, Howe and colleagues have studied the vibrations resulting from several different tasks in both virtual and teleoperated environments. Kontarinis and Howe[6] investigated the use of tactile displays for conveying task-related vibrations in both types of environments, using a haptic display augmented with a voice coil motor to provide high-frequency vibrations. For teleoperation, they successfully used a two-fingered master-slave system to perform tasks to identify damaged ball bearings, puncture membranes, and assemble a peg in a slot. In these experiments, the vibrations were measured with an accelerometer on the master fingertip and directly fed back through the voice coil motor. Using a similar feedback mechanism, teleoperation experiments were performed by Dennerlein and Howe[3]. Piezoelectric sensors embedded in the grippers of a six degree-of-freedom Schilling underwater assembly robot were used to sense vibrations, which were fed directly back to the user.

For vibration feedback in virtual environments, there is no simultaneous vibration measurement, thus a model is used in order to determine what should be displayed. Wellman and Howe[14] modeled the vibrations resulting from tapping on surfaces of different stiffness and played them back to observers through a force feedback device augmented with a voice coil motor. They also compared the ability of humans to distinguish between surfaces of different stiffness in real and virtual environments, finding that the virtual environments worked almost as well as the real ones. Okamura, et al.[9] expanded this work in surface stiffness and also considered the tasks of puncture and patterned texture stroking with a stylus. Vibration models were obtained for these tasks and assembled into a vibration waveform library. This work used the haptic display itself, rather than an additional voice coil, to display the vibrations.

Hard surfaces may also be displayed using other algorithms. The typical stiffness model used for surfaces simply outputs a force proportional to and in the opposite direction of penetration into the surface. Salcudean and Vlaar[12] use an impact energy dissipation method to make surfaces quickly damp out motion and thus appear stiffer. High damping upon impact provides a sudden spike of force similar to that which occurs in real environments. Ullrich and Pai[13] studied the mechanics of how vibrations arise when two surfaces collide. However, so many parameters are involved in this computation that it would be difficult to extrapolate a vibration feedback algorithm from this data.

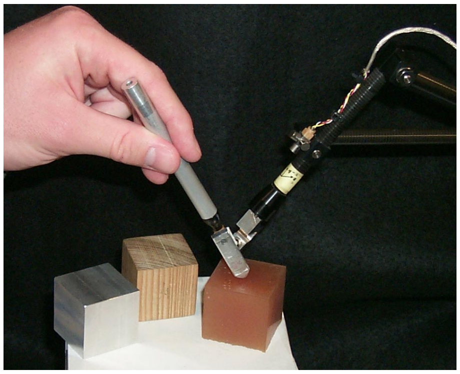

Figure 1. Tapping on material samples with an instrumented stylus.

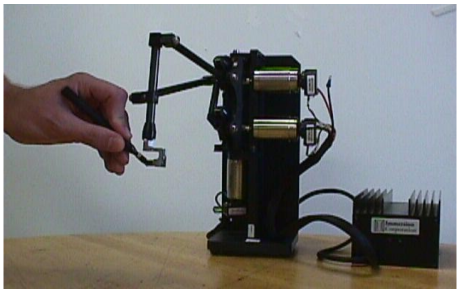

Figure 2. The 3GM haptic interface.

Much work has also been done in the design of haptic displays to make them stiffer and thus able to display harder surfaces[8]; this is a mechanical solution. In order to evaluate the capability of haptic displays to provide vibration feedback, one can consider several methods of characterization. Moreyra and Hannaford[8] created a metric called the Shape Deformation ratio that may be used to compare the bandwidth and stiffnesses of different haptic devices.

2 BUILDING VIBRATION FEEDBACK MODELS

The first experiment presented in this work was performed to develop a model and select model parameters for the vibrations resulting from tapping on surfaces. Previous research in using vibration feedback to distinguish between surfaces has shown that unless materials differ significantly in hardness or stiffness, humans cannot tell them apart even when tapping on real materials. Thus, in this set of experiments, we considered three materials with disparate stiffness: Urethane RTV rubber, wood, and aluminum. The samples used were solid 3.5 cm cubes of the materials, as shown in Figure 1.

The main experimental apparatus consisted of a six degree-of-freedom desktop haptic device with three degrees of freedom of force feedback and position/velocity sensing. Figure 2 shows this device, the 3GM from Immersion Corporation[5].

2.1 Taking Vibration Data

For taking vibration data, the stylus at the end-effector of the haptic device was instrumented with a tri-axial Kistler 8694M1 500g accelerometer and an aluminum extension to provide a good tapping surface (Figure 1). With the extension, the tapping stylus ends in a blunt point, approximately 1 mm in diameter. The diameter was measured by inking the tip, pressing it onto a piece of paper, and taking the diameter of the resulting mark. Position, velocity, and acceleration data were taken at 25kHz using a Pentium 166Mhz personal computer. The accelerometer was mounted on the haptic device near the endpoint.

The method of data collection in these tapping experiments was similar to that by Okamura, et al.[9], except here the data is taken with the haptic device itself rather than a different position-sensing device. This is important for haptic display because it allows direct comparison between the acceleration data from tapping on real and virtual surfaces.

2.2 Modeling

When modeling the vibrations resulting from tapping, there are several issues to consider. One is that the vibrations will be displayed using a computer program that takes time to calculate the necessary force output. Thus, the model should be simple enough that these calculations can be done quickly. Alternatively, table look-up or other means can be used to send out complicated waveforms with minimal computation time. Another consideration is the necessary accuracy of the models. Vibration frequencies much higher than 1kHz are not easily sensed by humans[10], so it may be unnecessary to display vibrations with such detail.

Looking at the vibrations resulting from tapping on each material, the same fundamental frequency is identified using a Fast Fourier Transform (FFT) despite differences in tapping speed. One anomaly in the data was that at very high-speed taps, the fundamental frequency shifted. (For example, rubber shifted from 18Hz to 25Hz). This can be explained by unmodeled nonlinearities in the system. When a nonlinear system is excited with different amplitudes of input, the frequency of the output may change. One example of such a case with vibrations is the analysis of musical instrument tones[11]. With linear systems, a change in input amplitude will only cause a change in output amplitude, not output frequency. There was also a characteristic decay rate for each material. Amplitude linearly increased with impact velocity, with a slope that remained constant for each material.

The vibration feedback model developed from the tapping data is a decaying sinusoidal waveform:

$$Q(t) = Ave^{-Bt}\sin(\omega t) \tag{1}$$

The estimated model parameters are A, the slope of the amplitude with respect to velocity, B, the decay rate of the sinusoid, and ω, the sinusoid frequency. Figure 3 shows the decaying sinusoid model fit to experimental data for tapping on wood.

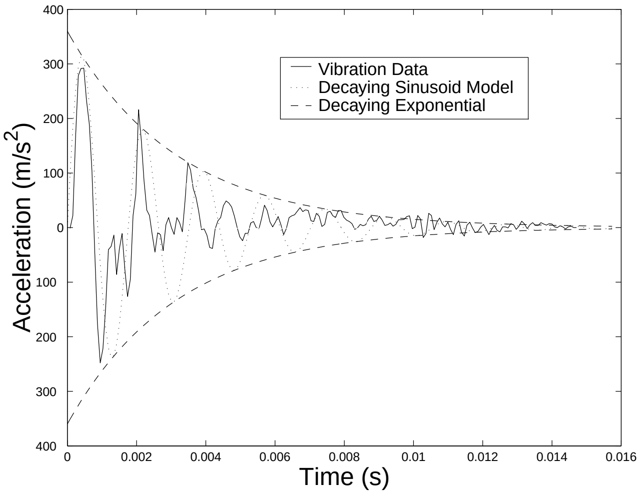

Figure 3. The decaying sinusoid model fit to experimental data for tapping on wood.

Material	A (s^{-1})	B (ms^{-1})	ω (Hz)
Rubber	-116.7	40	18
Wood	-1500	600	592
Aluminum	-100,000	1500	1153

Table 1. Modeled vibration parameters.

At the end of this analysis, we obtain vibration waveform models that describe the experimental conditions. The models cannot necessarily be abstracted or extended to other sizes and shapes or new materials. For example, no consistent relationship between modulus of elasticity and vibration frequency has been found. The shape of the object also has an effect: for example, an aluminum block will result in a different vibration than a thin aluminum sheet. Another important consideration is the material and geometry of the stylus, which is why data is taken with the same device that is used to display the vibrations.

Table 1 lists the parameter values for the three materials tested. Using these parameters to display vibrations, the model did not feel as realistic as possible, in part due to the inability of the haptic device to accurately display the desired waveform. An analysis of the haptic device using frequency methods was used in order to determine the physical basis of this phenomenon.

3 DISPLAYING THE MODELS ON A HAPTIC DEVICE

A frequency analysis of the haptic display demonstrated that it was unable to exactly replicate the desired real vibration waveforms.

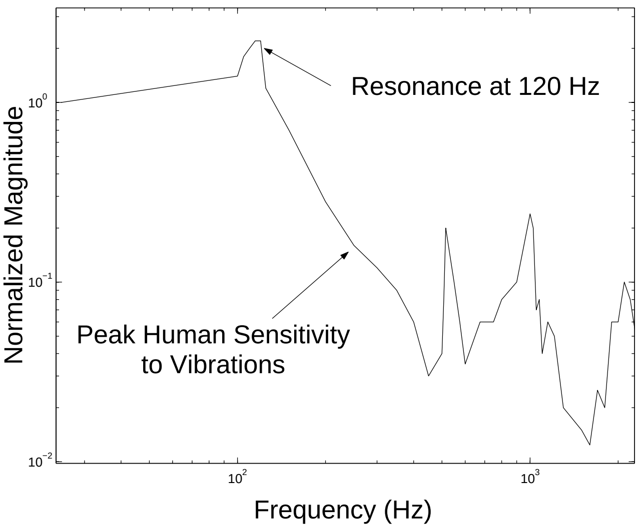

Figure 4. Normalized magnitude response of the 3GM haptic device.

3.1 Characterizing the Haptic Device

In order to determine the characteristics of the given haptic display, a frequency response of the device was obtained. The equipment consisted of a signal generator, an oscilloscope, and the 3GM haptic device with accelerometer as described previously. The signal generator was used to drive individual motors on the 3GM with sinusoidal waves of varying frequencies. The accelerometer was attached to the end-effector of the stylus and used to measure the acceleration of the haptic display. The oscilloscope was used to compare the input signal from the signal generator to the output of the accelerometer in order to determine the ratio of output magnitude to input magnitude.

Figure 4 shows the normalized magnitude response of the motor primarily responsible for displaying vibrations during tapping in the virtual environment. From this figure, it is apparent that the ability of the haptic display to display desired magnitude forces falls off as frequency increases. Unfortunately, this fall-off occurs at frequencies lower than those indicated in the vibration models (Table 1). This frequency response could be used to compensate for lower acceleration at higher frequencies, but was this technique was not used in the experiments presented below.

3.2 Haptic Display Parameters

Since it was not viable to exactly replicate the original vibration models obtained from experimental data, new models were necessary. These models used original vibration models as initial guidelines, scaled based on the given hardware and subjective evaluations.

The sensitivity for human vibration sensing ranges from DC to over 1kHz, with peak sensitivity around 250Hz[1]. Frequencies above this peak sensitivity likely contribute to the overall feel of a haptic simulation, however, the haptic display is not able to correctly implement high frequencies. Thus, we acknowledge limitations in the haptic display by limiting the haptic models to frequencies not much higher than 250Hz.

In order to scale down the vibration parameters for use with a haptic display, the following procedure was used. The frequency parameters of the model were decreased proportionally so that the scaled models included frequencies that could be displayed on the haptic device and the highest frequency (for aluminum) was near the peak sensitivity of human haptic sensing. Once the new frequency parameters were chosen, the decay rate and slope parameters were examined. By evaluating the simulated models as displayed by the haptic device, new values for decay rate and slope parameters were obtained, scaled proportionally for each material. These subjective evaluations were performed by the experimenters. Figure 5 depicts the process for obtaining the models from the measured vibration data, where the scaled model is the third block in the diagram.

This scaling process certainly distorts the vibrations; linear, proportional scaling of the different parameters may not be the way that human vibration sensing is scaled. In addition, the model is no longer completely reality-based after parameter scaling. Thus, further experiments were used to explore a region of parameter combinations near the scaled model to find the most realistic (local) parameter values.

4 IMPROVING MODELS WITH PSYCHOPHYSICAL EXPERIMENTS

In order to ascertain which simulated vibration model parameters felt most realistic, psychophysical experiments were performed to compare the realism of different parameter combinations.

4.1 Experiments

The experimental equipment consisted of the 3GM haptic device, blocks of real materials (rubber, wood, and aluminum) for simulated model comparison, and a control computer. A factorial design was used to determine the most realistic local model parameter values[2]. This method entailed choosing two values for each of the three parameters to be examined: frequency, decay rate, and amplitude slope (multiplied by impact velocity in the model). Each parameter is associated with a high and a low value, which were chosen to bound the value determined for the scaled model. These high and low values were chosen so that the difference between the models with the different parameter sets was above the Just Noticeable Difference (JND) of the experimenter. In order to test all combinations of the above parameters, 8 (2^3) different sets of parameters were displayed for each material. Figure 6 shows the parameter combinations tested for wood.

There are inherent limitations to this method, as it is possible that the “best” parameter combination lays outside the tested space, or even within its boundaries but not at the vertices. A series of factorial experiments would be needed to obtain a more accurate estimation of the most realistic parameter values. Other

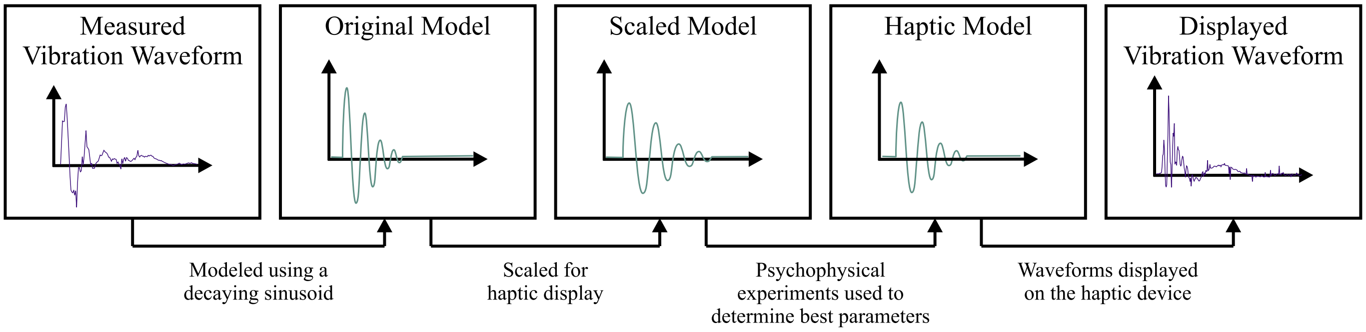

Figure 5. Measured and modeled vibration waveforms.

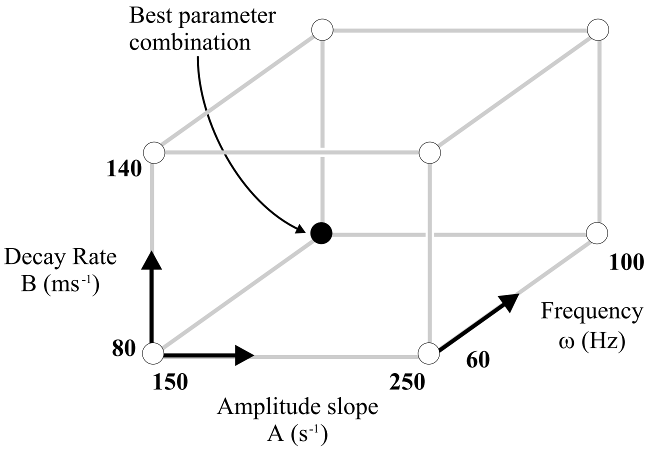

Figure 6. Factorial experiment design for wood. The black circle shows the best combination of parameters selected in the psychophysical experiments.

optimization methods, such as response surface methodology[7], could be used to perform a more comprehensive study. However, these methods would be difficult to implement because of the number of experiments required and operator fatigue.

The algorithm for displaying the vibrations is shown in Figure 7. The first plot shows the position of the end-effector of the haptic display as it approaches, penetrates, and leaves the virtual surface. The velocity at the moment of impact is measured. The stiffness force of the wall is displayed by measuring the surface penetration (Δz) and displaying a proportional force. For all the materials, the same stiffness k was used. (Thus, the materials differed *only* in the vibration feedback.) The vibration waveforms were superimposed on this stiffness force. The amplitude of the vibration waveform was determined by the impact velocity and the amplitude-velocity slope for the material. The appropriate decay rate and frequency parameters were also selected for the particular material being displayed.

Nine subjects were asked to evaluate the eight different simulated vibration models for each of the three materials. The subjects used the 3GM to control a cursor on the computer monitor,

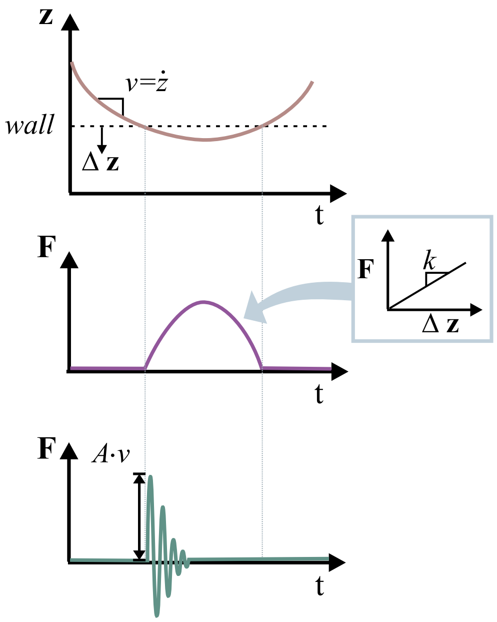

Figure 7. Algorithm for vibration and force feedback of the tapping task.

which included a virtual surface simulating the appropriate material. Subjects were encouraged to compare the simulated vibration models with blocks of real materials, in order to better evaluate the realism of the simulated model with respect to the actual materials. When the subjects interrogated a real material, they used the same haptic device to tap on the material. Subjects then rated each of the eight models for a material on a scale of 1 to 10 (least to most realistic). Each subject was presented with a random order of virtual materials and parameter combinations for each material.

4.2 Results

The experimental data consisted of ratings for each of the eight simulated models (the vertices of the box in Figure 6) for each material. In order to reduce the data to a meaningful form, each of the subjects' rating sets (different parameter combina-

Material	A (s^{-1})	B (ms^{-1})	ω (Hz)
Rubber	-240	60	30
Wood	-150	80	100
Aluminum	-300	90	300

Table 2. Best vibration parameters determined by psychophysical experiments.

tions for a given material) were normalized to have an average of 0 and a standard deviation of 1. This was necessary in order to compare the data of different subjects, who all had different means and variances. The results of the experiment (Table 2) show the best parameter values for the three materials.

The factorial experiment design also allowed calculation of the most significant parameter for each material. Using Analysis of Variance (ANOVA), it was found that decay rate was most significant in correctly displaying rubber, frequency for wood, and slope for aluminum. Different parameters, then, affect the display of different materials with varying levels of significance. This was an interesting result, and future work is planned to examine the source of these differences.

Qualitative subject responses revealed that rating the realism of the models was a difficult task. Several of the subjects commented that they did not feel any difference between some of the models. A larger difference between the low and high values for each parameter might have resolved this problem. Also, the number of models rated by each subject (a total of 30) was high and resulted in subjects becoming fatigued towards the end of the experiment. Subjects were allowed to take as much time as they desired, but there were no enforced breaks. Most subjects completed the experiment in approximately 20 minutes.

5 TESTING VIBRATION FEEDBACK MODELS

Based on the last set of experiments, the “best” model for each of the three materials (rubber, wood and aluminum) was determined. To verify that that these models were effective in a virtual environment, another psychophysical test was performed.

5.1 Experiment

In this experiment, subjects were shown a single virtual environment with three different surfaces, each using the vibration feedback model for rubber, wood, or aluminum, in random order. The virtual environment was displayed visually on the computer monitor and haptically with the Immersion 3GM, using the algorithm shown in Figure 7. The best values determined in the previous section were used for the vibration waveforms. The subjects were informed that there was only one of each material and were told to tap on the surfaces in order to distinguish between them. They were also provided with real samples of each material to tap on for comparison. Each subject did this identification experiment three times. The 14 subjects ranged in age from 11 to 35 and in experience level from novice to expert.

5.2 Results

As shown in Table 3, most of the experiments resulted in correct identification of the three materials (83.3%). A “correct” answer occurred when the subject correctly identified all three of the materials. Correct identification of one material but switching the other two resulted in an incorrect answer. Each of the 14 subjects performed the experiment 3 times, resulting in a total of 42 experiments. In the post-experiment questionnaire, many of the subjects revealed that they felt they were learning as they progressed through the three experiments. One can see that when only the third experiment with each subject is considered, the success rate is slightly higher (85.5%). It is also notable that the difference in performance between novice and expert subjects was not large (88.9% success for experts and 79.2% for novices).

The most common error in surface identification was mistaking wood and rubber. Many of the subjects felt that the high frequency “ringing” of the aluminum model was a clear indication of a metallic material. In contrast to the realism rating experiment, many of the subjects for this test felt that it was straightforward. This is expected, as the task of distinguishing between different materials is much easier than rating very similar models of the same material. When asked to comment on the realism of the surfaces, a few subjects noted that the vibrating surfaces felt “active” and thus unrealistic. However, many subjects felt that, while the virtual materials did not exactly feel “realistic”, they did feel more so in comparison with a virtual surface without vibrations.

6 CONCLUSIONS

This work showed that reality-based models of vibrations can be used to enhance virtual environments. The following points address the primary questions posed in Section 1.

- An impact vibration is similar in shape to a decaying sinusoid. Model parameters can be determined by analyzing the FFT, ratio of maximum amplitude to impact velocity, and the envelope of the measured waveform.
- The haptic device should have a significant response for frequencies at which humans have vibration sensitivity. The vibration waveforms for some materials, such as aluminum, have high frequencies and cannot be accurately displayed or sensed. High waveform amplitudes are also impossible to display with conventional haptic devices. Thus, when waveform parameters are out of the range for practical display, the waveform is scaled down.
- Vibration feedback models can be made to feel more realistic through parameter value selection using psychophysical experiments. When the vibration waveforms were modified from the original model to the scaled model, initial testing showed that the selected vibration parameters were not nec-

Data type	Number of Experiments	Correct	Incorrect	Percent Correct
All Experiments	42	35	7	83.3%
Third Test only	14	12	2	85.7 %
Experts only	18	16	2	88.9%
Novices only	24	19	5	79.2%

Table 3. Experimental results for subject identifying different materials.

essarily the best for making the virtual surfaces feel real. Thus, psychophysical experiments were used, allowing subjects to rate various parameter combinations for realism.

- The vibration feedback was shown to be effective by the final experiment, in which subjects identified three different virtual materials that differed by vibration feedback alone. Many of the subjects felt that the vibrations enhanced the realism in comparison with surfaces without vibration feedback.

6.1 Discussion and Future Work

During this study of vibration feedback, many interesting issues arose which merit further discussion and also bring possibilities for future work.

6.2 Model Type and Experimental Conditions

Two fundamental concerns are whether the model used is optimal, and whether the model will change under different experimental conditions. There are many other possible models for vibration waveforms, including multiple sinusoids and wavelets. An important consideration is that the model used should be simple enough to be calculated in a servo loop running at 1 kHz or greater (a common servo rate for haptic displays). Look-up tables and other means may also be incorporated if a complex model is to be used.

The experimental conditions for taking tapping data, including the type of stylus (material and shape) and the geometry of the material being tapped, also have an effect on the shape of the vibration waveforms. Preliminary experiments showed that when a stylus is tapped in a cantilever position, the vibration waveforms are significantly affected. By tapping with the stylus perpendicular to the surface, the resonant frequency of the stylus is higher and has less effect on the resulting vibrations. It is also apparent that the geometry of the material being tapped will also result in different vibrations. For example, tapping on the hood of a car sounds and feels different than tapping on a solid cube of steel.

6.3 Displaying Vibrations With Haptic Device Limitations

Typical haptic display systems cannot display the stiffness of real surfaces, nor can they exactly display the desired vibration waveforms. Since some of the desired vibrations were out of the frequency and acceleration limits of the haptic device, the haptic models were scaled down from the original models. One might argue that there is no motivation for reality-based modeling if the parameters obtained from the original model are not used. However, it is important to perform this modeling to verify the shape of the waveform and get initial values for frequency, amplitude, and decay rate. Comparing the original parameters from different materials also provides information about the differences between the responses of various materials.

Even with the scaled down haptic model, the accelerations resulting from tapping on a virtual surface do not perfectly match the original model (as is apparent from a comparison of the last two blocks in Figure 5). Additional vibrations occur during display because of the dynamics of the haptic device. Despite the fact that the haptic display cannot display the exact vibrations that were modeled, there is evidence that the technique is effective (Section 5). The vibration feedback presented in this work provides cues that may be “good enough” for most applications.

Another consideration is the type of vibration. There is a classic debate whether the sensed property is force or displacement. Accelerations were measured during tapping on real materials, which were converted to forces that produce the same acceleration at the endpoint of the haptic display, given the mass of the device. However, when the user is in contact with the stylus, a combination of forces and displacements of the stylus are sensed by the cutaneous and kinesthetic sensors in the hand.

Another approach to improving the realism of virtual environments is to design haptic devices with the ability to display harder surfaces (for example, using brakes). However, the goal of this work was to increase realism only using algorithmic modifications, so that current haptic displays do not have to be augmented or redesigned.

6.4 Further Evaluation of Vibration Feedback

There are certainly more tests that can be done to verify the effectiveness of vibration feedback. For example, experiments should be performed to compare the vibration feedback algorithms presented in this work against other algorithms for making surfaces appear hard, such a impact damping[12]. The role of auditory feedback of vibration is also important. Preliminary experiments show that when auditory feedback is removed (by playing white noise), the task of distinguishing between some materials via haptic vibrations alone is difficult, especially for novice users. Although previous work[9] showed that materials can be distinguished even when auditory feedback is masked, further work should be done to determine the significance of this effect.

Vibration feedback is also useful in many other tasks. Although preliminary studies of texture stroking and puncture tasks have been performed[6, 9], these tasks and others warrant a more thorough investigation.

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