Lab 5 - Feedback Control

PRE-LAB
In this lab, you will be using PD feedback control to change the behavior of your haptic paddle. Recall the equation of motion you found back in PRE-LAB 3.

$$m_e\ddot{x} + b_e\dot{x} + k_e x = f_m + f_c$$

Let's ignore Coulomb friction for now and assume that there is no force from the motor. We get a classical second order homogeneous equation

$$m_e\ddot{x} + b_e\dot{x} + k_e x = 0$$

where

$$
\begin{aligned}
m_e &= J_s\left(\frac{1}{l_x}\right)^2 + (J_p + J_m)\left(\frac{r_s}{r_p l_x}\right)^2 \\
b_e &= \left(\frac{r_s}{r_p l_x}\right)^2 b_m \\
k_e &= -mgr_c\left(\frac{1}{l_x}\right)^2
\end{aligned}
$$

a. Using data from previous labs, calculate m_e, b_e, and k_e. Assume that the inertia of the pulley J_p is small enough that you can ignore it.
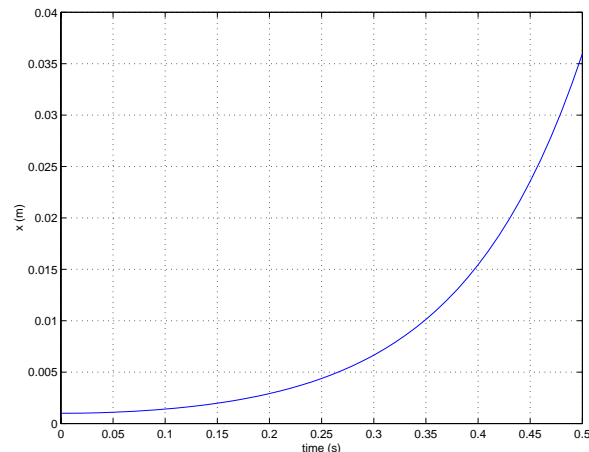**Result:**

$$
\begin{aligned}
m_e &= 0.0191 \quad \text{kg} \\
b_e &= 0.0157 \quad \text{N s/m} \\
k_e &= -1.50 \quad \text{N/m}
\end{aligned}
$$

(Your answers may differ)

b. Plot $x(t)$ for the initial conditions $x(0) = 0.001$ and $\dot{x}(0) = 0$. What is going on here?
Result: Because the stiffness is negative, the system is inherently unstable. It rises slowly at first because there is damping and inertia in the system, but eventually it goes to infinity.

1

When your haptic paddle is attached to an amplifier circuit and a computer equipped with A/D and D/A boards, you can apply a current to the motor that will change the input to the system f_m. Because you can now apply any (reasonable) f_m you want, let's see how we change the behavior of the system. Since we can use the hall effect sensor you calibrated in the last lab to measure the position and velocity [1] of the handle, we can set

$$f_m = -c_1 \dot{x} - c_2 x$$

where the feedback gains c_1 and c_2 are anything you choose. Ignore Coulomb friction for the rest of this problem.

c. Write the new equation of motion for this system, grouping the terms with x and its derivatives together.
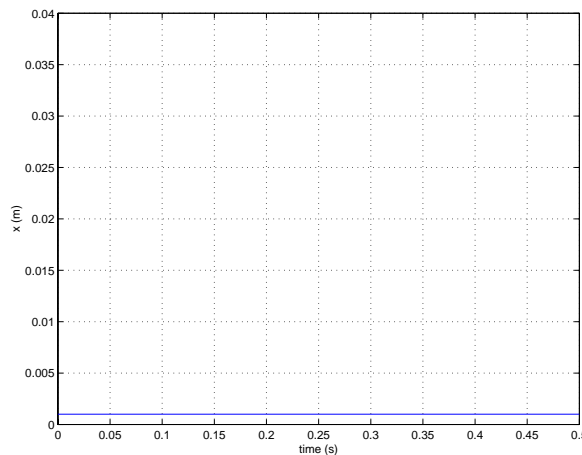**Result:**

$$m_e \ddot{x} + (b_e + c_1)\dot{x} + (k_e + c_2)x = 0$$

d. Let $c_1 = 0$ and make the proportional feedback exactly cancel out the equivalent stiffness of the system. What is the value of c_2? For this case, plot $x(t)$ for the initial conditions $x(0) = 0.001$ and $\dot{x}(0) = 0$. What is going on here?
Result:

$$c_2 = mgr_c \left(\frac{1}{l_x}\right)^2$$

Because there is no stiffness and there is some damping, it doesn't want to go anywhere. The position doesn't change.

e. Choose value of the feedback gains c_1 and c_2 to give a response with $\zeta = 0.7$ and $\omega_n = \pi$.
Result:

$$c_1 = 0.0683 \ \text{N s/m}$$
$$c_2 = 1.69 \ \text{N/m}$$

(Your answers may differ)

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[1] The velocity is measured in the software by taking several position measurements and dividing by time. For more details, feel free to poke around the C++ source code, which is posted on the web page

LAB

The purpose of this lab is to control your haptic paddle with a PD feedback control law. You will give your paddle initial conditions and step inputs and record the response. Note: Bring a diskette to the lab with you.

In the lab, you will find two amplifier circuits attached to the a computer with a A/D (Analog to Digital) and D/A (Digital to Analog) card. The A/D part of the card is used to read the voltage from your sensor and convert it to digital information in the computer. The D/A part of the card is used to convert information in the computer to an output voltage. This output goes to the amplifier circuit, which then powers your motor. The schematic below shows the lab setup.
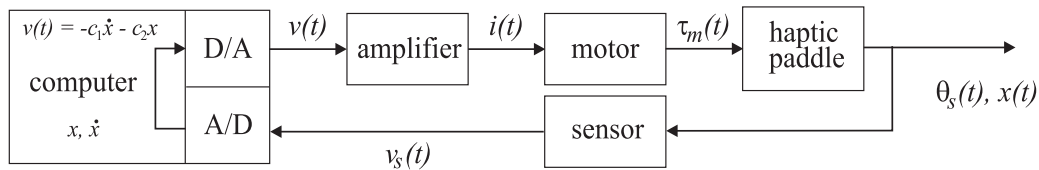
1. If you haven't yet finished the assembly, complete it now. Use the nylon screws and washers to anchor the string to the sector pulley. The string should be wrapped 3 to 4 times around the motor pulley. Use the cantilevered part of the sector pulley to get good string tension: pinch it closed while you tighten and attach the string, then when you let go, the string will tighten.

2. Hook up your sensor (but not your motor yet) to the +5V, Ground, and Signal wires for your station. Looking at your sensor from the top, with the leads facing you, the leads are +, ground, and signal from left to right. Make sure the sensor power supply is on.

3. Run the program `c:\me161\haptic.exe`. From the main menu, select the `Test Calibration` option by pressing `t`. You will be asked to enter your station number and group number. (We have already entered in your coefficients A, B, C, and D for you.) The program will display the computed x position in millimeters as you move the handle back and forth. It should appear to vary smoothly and linearly with handle position. If not, something is wrong with the calibration. See us for help before going further. Hit `q` to exit the calibration tester.

4. Now you are in the main menu again. Hit `r` to run the feedback program. Use the arrow keys on the keypad (Num Lock should be on) to get to the correct line number for your station. Move horizontally to get to the second column, and enter in your group number. In the third column, you can select your station to be `on` or `off` by pressing `+` or `-`. Start with the station off. Now hook up the motor leads.

5. Move to the fourth and fifth columns and press the `+` and `-` keys to increase and decrease the values of c_1 and c_2. First try for a modest c_2 of about 5.0 and keep $c_1 = 0$. Now turn your station on, while holding the handle of the joystick. Pull the handle to one side. It should try to come back to center, as if a weak spring were attached to your pulley. (If instead it moves quickly all the way to one side, the T_m is inverted; reverse the leads to the motor.) You are feeling $k = k_e + c_2$! If the sector pulley does not center itself when you release it, then your device probably has too much friction. Try a bit higher c_2, say 8.0. If it still does not work, see us.

6. Now, pull the handle to one side and let go. You should see a classic underdamped free-motion kind of response. You can change the value of c_2 to make it respond well. In this case, the only damping is the natural damping in the system. Take a plot of this by going to your station line and typing `t` just before you release the handle. You'll get three seconds worth of data after you hit the key. Then

you will prompted for a filename; choose a creative one that won't likely be written over by another team. Enter it without the extension, a `.m` will be added by the computer program.

7. Now put in a step function. The feedback control law will become

$$f(t) = -c_1\dot{x} - c_2 x + A$$

where A is the value of the step input. Move your cursor to the sixth column and use `+` or `-` keys to change the magnitude of the step input. Type `a` to apply the step, and `r` to remove it. How big of a step function does it take for the handle to move noticeably? (The lower, the better - this test basically tells you how much static friction you have; the higher the friction, the greater the step function needs to be before the handle will move.) Record this minimum value for your write up. Now put in a larger step function, but not so much that the handle reaches it's limit of motion and knocks into the stops. Observe the amount of oscillation. You can try a negative step, and you can also see motion when you remove or change the step function. Hit your station number again to record data for a step response. Use a different filename from the last one and record the step magnitude.

8. Now remove the step function and increase c_2 about as far as you reasonably can, that is until the system starts buzzing or chattering or showing other signs of incipient instability and general unhappiness when you move the handle around a little bit. This may correspond to a c_2 of 12 or more depending on your system. If you pull the handle all the way to one side, you will feel a respectable force trying to pull you back to center. (The motor might slip, especially if your string is loose. If this is the case, turn off your station using the computer program and tighten the cable or add more wraps.) Now, with this higher stiffness, do the step function again and take data, using a new file name. The system should be noticeably more oscillatory with a higher value of c_2. Why is that?

9. Now keep your healthy value of c_2 and experiment with adding a bit of c_1. If you add too much you'll get chatter because of the inaccuracies in estimating velocity from a digital position signal. Still, a bit of c_1 may help to reduce oscillations.

10. Now experiment what happens with small negative values of c_1. Explain in your write up what is going on from a physical standpoint.

11. Do the same thing with small negative values of c_2. Explain in your write up what is going on from a physical standpoint.

12. Disconnect your motor leads. When you quit the program, you can copy your data files onto a diskette for answering the questions below. Don't quit the program without first warning anyone else who is also testing their device!

Questions:

1. From part 7 above, what was the minimum value of step required to move the haptic paddle?

2. Use `matlab` to look at your plots from steps 6, 7 and 8 above. There is an example `matlab` script on the web page. Choose one set of data and estimate (i) the closed loop natural frequency ω_d and (ii) the damping parameter ζ.

3. Plot your theoretical (using the ω_d and ζ above) and actual response on the same plot for your data of choice. Think about the initial conditions and don't forget to make both start at the same time.

4. Can you identify how Coulomb friction affects the response?

5. Explain why the system is more oscillatory for a high value of c_2 (see step 8).

6. As requested in steps 10 and 11, explain what happens when you make c_1 and c_2 negative.