Lab 2 - Bifilar Pendulum

PRE-LAB

In class, we will do a lab experiment to determine the inertia of the handle/sector pulley assembly of your haptic paddle. We describe the how to obtain the equation of motion of the bifilar pendulum system in a handout from Steidel (*An Introduction to Mechanical Vibrations*, 3rd ed., pages 60-61).

a. Go through the steps to obtain the bifilar pendulum equation of motion. (Show all your work: drawing the schematic, drawing the FBD, getting the forces, then applying Newton's law.)
Result:

$$\ddot{\theta} + \frac{mgD^2}{4Jh}\theta = 0$$

b. If the second order equation is of the form $\ddot{\theta} + {\omega_n}^2\theta = 0$, what is the natural frequency f_n of the system in Hz $(= \frac{1}{sec})$? Solve for f_n of the bifilar pendulum.
Result:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{mgD^2}{4Jh}}$$

c. Solve for the inertia J in terms of f_n.
Result:

$$J = \frac{mgD^2}{16\pi^2 {f_n}^2 h}$$

d. Using the bifilar pendulum experiment, you determined the inertia about the center of mass. When your haptic paddle is in use, however, the handle/sector pulley assembly will rotate about a point at a distance r_c from the center of mass. From the *parallel axis theorem*, the inertia J_o about the center of rotation can be calculated by $J_o = J + m{r_c}^2$. Write the equation for J_o in terms of measured quantities.
Result:

$$J_o = \frac{mgD^2}{16\pi^2 {f_n}^2 h} + m{r_c}^2$$

e. Determine the error in your result of J_o based on the error in your measured quantities. You can do this by using the standard formula for the variation in a function y of several toleranced variables, x_i:

$$\delta y \cong \sqrt{\left(\frac{\partial y}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial y}{\partial x_2}\delta x_2\right)^2 + \cdots + \left(\frac{\partial y}{\partial x_n}\delta x_n\right)^2}$$

Your toleranced variables are m, D, f_n, h, and r_c. If you have trouble with taking the partial derivatives of J_o with respect to these variables, please come see one of the instructors for assistance.
Result:

$$\delta y \cong \sqrt{\left(\frac{\partial J_o}{\partial m}\delta m\right)^2 + \left(\frac{\partial J_o}{\partial D}\delta D\right)^2 + \left(\frac{\partial J_o}{\partial f_n}\delta f_n\right)^2 + \left(\frac{\partial J_o}{\partial h}\delta h\right)^2 + \left(\frac{\partial J_o}{\partial r_c}\delta r_c\right)^2}$$

where

$$\frac{\partial J_o}{\partial m} = \frac{gD^2}{16\pi^2 {f_n}^2 h} + {r_c}^2$$
$$\frac{\partial J_o}{\partial D} = \frac{2mgD}{16\pi^2 {f_n}^2 h}$$
$$\frac{\partial J_o}{\partial f_n} = -\frac{mgD^2}{8\pi^2 {f_n}^3 h}$$
$$\frac{\partial J_o}{\partial h} = -\frac{mgD^2}{16\pi^2 {f_n}^2 h^2}$$
$$\frac{\partial J_o}{\partial r_c} = 2mr_c$$

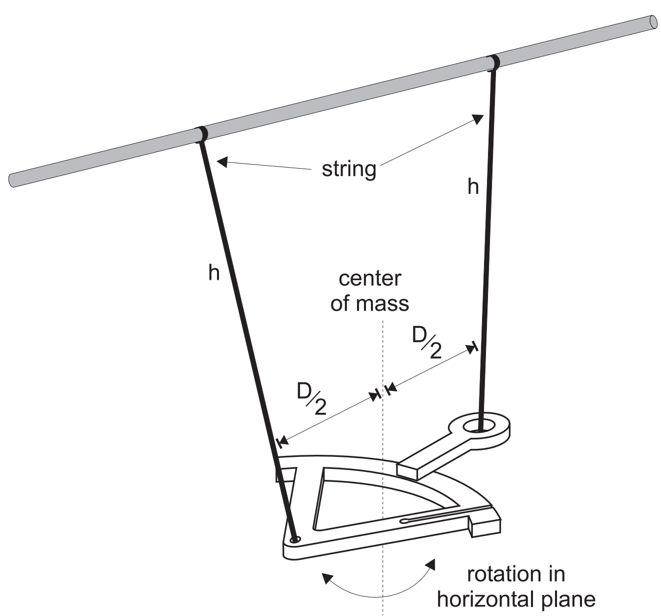
LAB

In this lab, you will use the Bifilar Pendulum method (see the Steidel handout) to determine the inertia of the handle/sector pulley assembly of your haptic paddle. We'll do this experiment in class, divided by lab groups. Come to the front of the room and pick up:

- A metal bar
- Some string
- A ruler

You will also need to use:

- A scale (at the front of the room)
- A digital watch or a watch with a second hand
- A lab partner who can count to at least 20



Please do the following steps. When taking measurements, also record the estimated error in the measurement.

- Find the approximate center of mass and identify two points you can use to hang the assembly. The points should be opposite each other and a distance $\frac{D}{2}$ from the center of mass. You can use the non-permanent pens at the front to mark your points. Record the distance D in cm.
- Record the distance r_c (in cm) from the center of the mass to the hole that will be the center of rotation when the haptic paddle is assembled.
- Use the scale to get the mass m (in grams) of the assembly.
- Using the string and the metal rod provided, set up the experiment as shown in the diagram above.
- Measure the length h (in cm) of the string used to attach the assembly to the rod.
- Deflect the pendulum (the assembly) and measure the natural frequency. Have one person count the number of oscillations while another person records the time. The frequency is the number of revolutions divided by the total time. The longer the time period, the better your estimation of frequency will be, however, you want to be sure that your pendulum is oscillating enough to see and count easily.
- Check the lab session sheets at the front of the room to determine your lab group number. After you do the pre-lab problem in the homework, get together with your lab partners and answer the lab questions below. Put your lab group number on your lab write-up.

Questions:

1. What is the inertia J_o? (This is the inertia about the center of rotation when the assembly is in use in the haptic paddle, not the center of mass). Use your solution to the pre-lab question in the homework (2.4d).
2. Make a chart showing the variables you used to calculate the inertia and the error in each variable.
3. Using your solution to the pre-lab question in the homework (2.4e), determine the error in your measurement of J_o. What is the percent error in J_o?