Lab 3 - Equivalent Inertia

PRE-LAB

In this problem, we will find the equation of motion for the haptic paddle and determine the equivalent mass, stiffness, and damping of the system. An assembled haptic paddle is available in the lab for inspection, and you will be assembling most of your device in the laboratory assignment. An unlabeled schematic of the device is shown below:

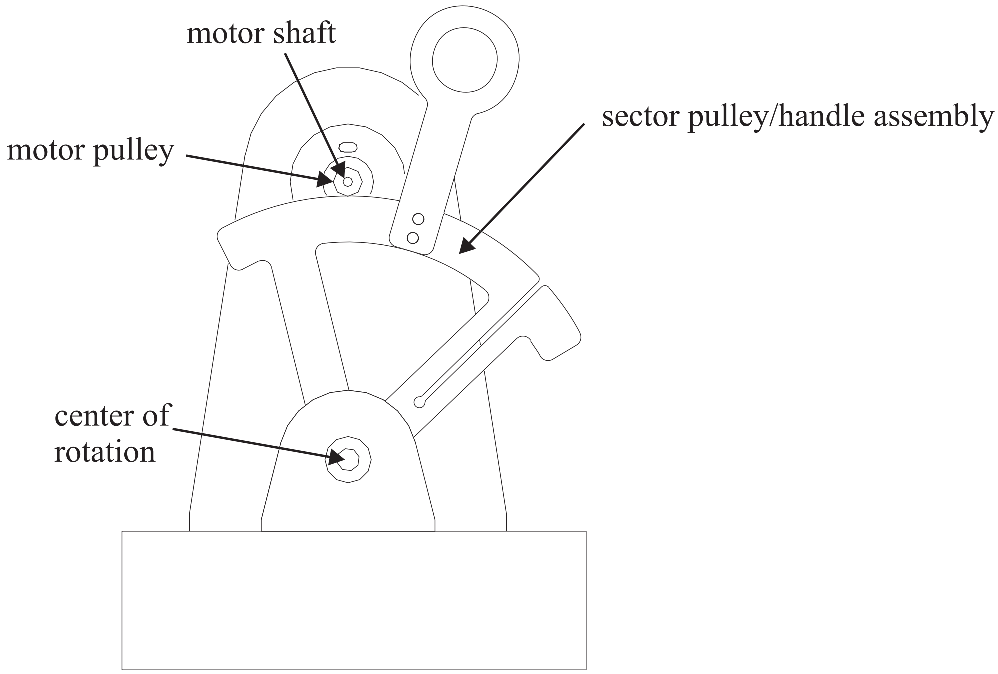


Figure 1: Haptic Paddle Schematic

- r_c is the distance from the center of rotation to the center of mass of the sector pulley/handle.
- l_x is the distance from the center of rotation to the center of the circular part of the handle.
- r_s is the distance from the string on the sector pulley to the center of rotation.
- r_p is the radius of the motor pulley.
- The sector pulley/handle has a mass m.
- There is an applied torque T_m from the motor on the motor pulley.
- There is a static friction torque T_c in the motor.
- The motor pulley and sector pulley rotate together because they are coupled by a cable.
- x, pointing to the right, represents the position of center of the handle from vertical as measured along a circular arc.
- θ_s represents the angle of the sector pulley from vertical. The positive direction is clockwise.
- θ_m represents the angle of the motor from vertical. The positive direction is counter clockwise.

a. Although there are several variables described above (x, θ_s, and θ_m), this system has only one degree of freedom. Find the kinematic relationships between (1) x and θ_s and (2) x and θ_m.
Result:

$$x = l_x \theta_s \qquad x = \frac{l_x r_p}{r_s} \theta_m \tag{1}$$

b. Redraw the schematic above and label it with the lengths, forces, and coordinates described above.

c. Using x as the dependent variable, calculate the equivalent mass m_e for the haptic paddle using the kinetic energy method.
Result:

$$m_e = J_s \left(\frac{1}{l_x} \right)^2 + (J_p + J_m) \left(\frac{r_s}{r_p l_x} \right)^2$$

d. For equivalent damping, assume that the only damping in the system is in the motor, b_m. Use power dissipation method to find b_e in terms of l_x, r_s, and r_p.
Result:

$$b_e = \left(\frac{r_s}{r_p l_x} \right)^2 b_m$$

e. Now compute the equivalent stiffness k_e using the potential energy method.
Result:

$$k_e = -mgr_c \left(\frac{1}{l_x} \right)^2$$

f. There are two torques applied in this system, the motor torque T_m and the Coulomb friction in the motor T_c. Find the inhomogeneous terms in the differential equation for x due to T_m and T_c, respectively.
Result:

$$\begin{aligned} f_m &= -\frac{r_s}{r_p l_x} T_m \\ f_c &= -\frac{r_s}{r_p l_x} T_c \end{aligned}$$

g. Write the equation of motion for the equivalent system, including the inhomogeneous term and equivalent mass, damping, and stiffness.
Result:

$$m_e \ddot{x} + b_e \dot{x} + k_e x = f_m + f_c$$

where m_e, b_e, k_e, f_m, and f_c are defined in parts d, e, f, and g.

h. Draw a schematic for the horizontal mass-spring-damper system that corresponds to this equation of motion. Label all elements and forces, and take x pointing to the right.

LAB

The purpose of this lab is to assemble some of the parts of your haptic paddle and get a feel for the equivalent inertia of the system. In the lab, you will find your kit in a bag with your group number on it. (A list of groups and members are posted in the lab as well.) Please give the instructor or TA a check for $30 made out to Stanford University before taking your kit. Write your group number on the check, please. If you decide not to keep your kit at the end of the course, and all the parts are still present, you will be refunded. Your kit doesn't contain all the parts yet, but you will have everything you need to complete this lab. Eventually, your haptic kit should contain the following parts (a * means that you will use the part in this lab):

- 1 Maxon motor, number 2326.948
- 2 stainless steel 4/40 screws for attaching the motor to the acrylic motor stand
- 1 motor pulley (looks like a thick-walled hollow cylinder)
- 7 acrylic parts
    * sector pulley
    * sector pulley stabilizer (triangle),
    * handle
    * small stand
    * big stand (for the motor)
    – hall effect sensor stand (0.5" cube)
    – magnet mount (0.4" × 0.4" × 0.5" with hole)

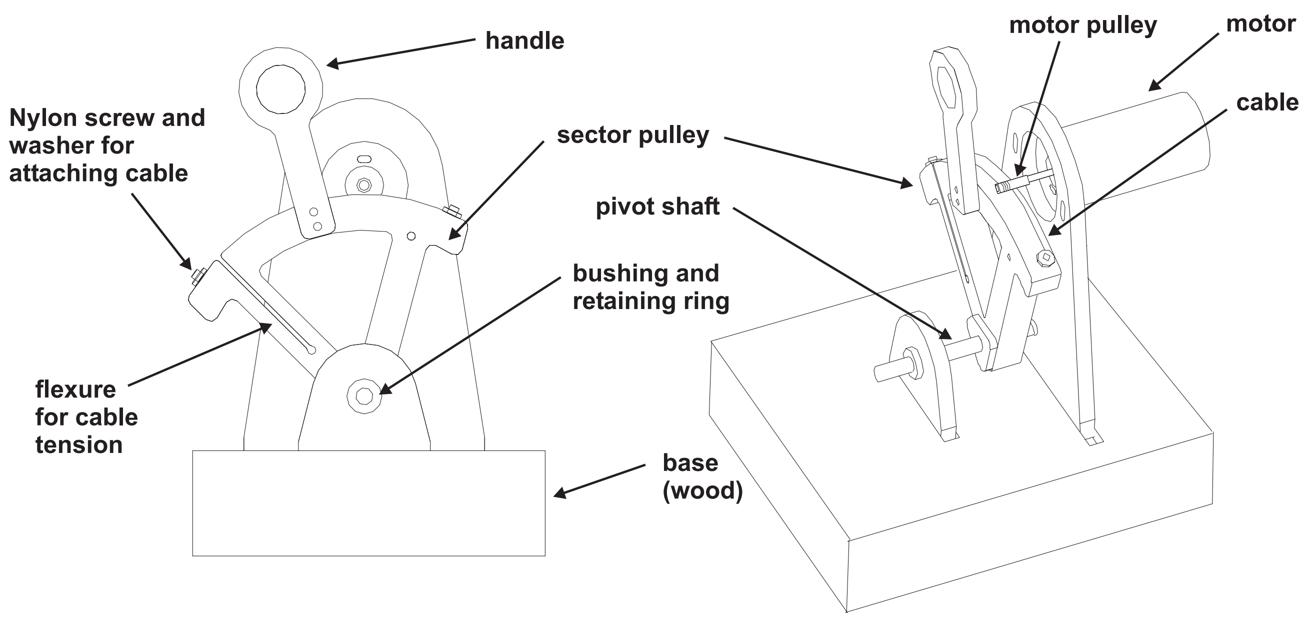
* 1 wood base
* 1 3" steel shaft
* 2 retaining rings
* 2 bronze "oil-less" bearings

- 2 nylon screws and 2 nylon washers
- 2 feet of Spiderwire cable

You should find in the lab:

- Loctite or super glue
- Acrylic cement
- Epoxy or hot glue gun and hot glue
- retaining ring tool
- an assembled kit for example
- a hand reamer in case the holes for the shaft are too small

The figure below shows the major parts of the device in two views:



Do the following steps to assemble your haptic paddle:

1. Attach the handle to the sector pulley using acrylic cement. Remember, with acrylic cement, a little goes a long way! Use the pair of holes and in each piece for alignment. You might want to borrow another group's shaft and use your own to put in the holes in order to get good alignment. Leave the shafts in the holes while it dries (about 5 minutes).
2. With acrylic cement, glue the sector pulley and the sector pulley stabilizer pieces together, using the shaft for alignment. The stabilizer should be on the same side of the sector pulley as the handle. The holes have a bit of a taper to them from the lasercam process, which is why you need the stabilizer. If the holes are too small to fit the shaft through, use the hand reamer in the lab to enlarge the holes. In the final assembled device, the shaft needs to rotate with the sector pulley, so try to keep a tight fit.
3. Place one bronze bearing in each acrylic stand. The holes have a slight taper, so place it in the direction that gives the tightest fit.
4. Glue the small and large acrylic stands to the wood base, using hot glue or epoxy (takes longer to dry). Again, use your shaft for alignment. Look at the example assembly to see which slot to use for each stand. The flanges of the bronze bearings should be facing the outside.
5. Now you are ready to assemble all the pieces on the shaft. The sector pulley goes in between the two stands. The small triangular piece (stabilizer) on the sector pulley should face the small acrylic stand. Later you might want to glue the sector pulley to the shaft using Loctite or superglue, but that will wait until you have attached the motor and found the correct spacing (in a future lab).
6. Use the retaining ring tool to place a retaining ring against the flange of each bearing. The sector pulley and shaft should rotate easily.

That's all the assembly for now. You'll be attaching the motor and the sensor parts in later labs.

Before you leave the lab, move around the handle of your partially assembled haptic paddle (with no motor attached) and then do the same with the fully assembled example. Notice the difference in effective inertia at the handle (the variable x in the pre-lab) between these two systems.

Questions:

1. The inertia of the rotor of your motor is 8.63 g-cm^2. What was the inertia of your sector pulley/handle assembly from the last lab? (Use the same units for both so you can compare).
2. The radius of the motor pulley r_p will be about 0.6 cm. The radius of the sector pulley from the center of rotation to the string r_s is 7.3 cm. What is the speed ratio $\frac{r_s}{r_p}$?
3. Based on your equation for equivalent mass m_e in the pre-lab, how much do the inertias of (1) the sector pulley and (2) the motor contribute to the equivalent mass? ($l_x = 12$ cm) Which one contributes more?
4. Does the speed ratio $\frac{r_s}{r_p}$ make the equivalent mass due to the rotor of the motor more or less? If the speed ratio was 1, then which part (the sector pulley/handle or the rotor of the motor) would contribute more to the effective mass?