Lab 4 - Motor Constants and Sensor Calibration

PRE-LAB

Consider a DC servo motor which is connected to a power supply. The motor inertia is J_m. The rotational damping in the motor is b_m. The motor produces a torque $T_m = K_m i$ where K_m is the motor's torque constant and i is the current from the power supply.

a. Write the mechanical equation of motion for this system in terms of θ. Rewrite it in terms of angular velocity, ω.
Result:

$$J_m\dot{\omega} + b_m\omega = T \tag{1}$$

b. Now consider the electrical part of the system. The motor coils have inductance L and a resistance R. The back emf of the motor is $V_m = K_v\omega$. Draw the circuit diagram for the motor when an external voltage $V(t)$ is applied. Then obtain the differential equation for $i(t)$ in response to the external voltage.
Result:

$$V(t) = iR + L\dot{i} + K_v\omega \tag{2}$$

c. Show that in the ideal no-load condition $i \to 0$ as $K_v\omega \to V(t)$ and that the stall torque approaches $\frac{K_m V(t)}{R}$ as $\omega \to 0$. Assume that $V(t)$ is constant and steady state has been reached.
Result:
If $i \to 0$, $K_v\omega \to V(t)$.
If $\omega \to 0$, $i \to \frac{V}{R}$ and $T = K_m i$.

d. Consider the case where L is negligible (a small motor with few windings). Combine equations from parts a and b to obtain a single differential equation relating ω to the input voltage $V(t)$.
Result:

$$\frac{J_m}{K_m}R\dot{\omega} + \left(\frac{b_m}{k_m}R + K_v\right)\omega = V(t) \tag{3}$$

e. If necessary, rearrange this equation so that the non-homogeneous term is a torque ($\frac{K_m V(t)}{R}$). Verify that the units of each term are torques (N-m). What is the equivalent rotational damping?
Result:

$$b_e = b_m + \frac{K_m K_v}{R} \tag{4}$$

LAB
The purpose of this lab is to measure various motor constants and calibrate your hall effect sensor. Note: Bring a diskette to the lab with you.

Motor: The motor constants we will calculate are: the torque constant K_m, the Coulomb friction T_c, the speed constant K_v. Note that we already measured the damping constant of the motor in Lab 1.

- To measure the torque constant K_m, apply a known current through the motor and measure how much torque it corresponds to. To do this, place your motor in the stand provided. Hook up the power supply to the motor with an ammeter in series. For the ammeter, you can use one of the Digital Multi-Meters (DMMs) found in the lab. To apply a known torque, use one of the pulleys (with a set screw), with string and weights provided in the lab. From the weight applied and the radius of the pulley, you can calculate the applied load. Wrap the string around the pulley attached to the motor, and increase the current from the power supply until it is just enough to balance the load. Do this for several different weights and obtain a torque vs. current plot. It should be approximately linear, and the slope is K_m.
- Compute the friction torque by determining the minimum amount of current required to spin the motor. Using the K_m you calculated earlier, you can determine the torque. Try spinning the motor from a standstill in both directions, and also try spinning it then lowering the current until it stops. You will get a range of values, representing static and kinetic friction.
- To measure the speed constant K_v, you need to measure the voltage versus speed. In order to do this, think of the motor as a generator. In usual operation, applying voltage/current to the motor causes it to spin. However, if the motor is spun from an external source, a voltage will be created across the leads. A motor with an encoder on a stand is provided in the lab to spin up your motor across a flexible coupling. The output of the encoder is a pulse for every revolution. Measuring the frequency of this pulse on the oscilloscope will give you the speed. Use a DMM across the leads of the motor to measure the voltage. Do this for several speeds, and obtain an approximately linear plot of voltage versus speed. The slope of this line is K_v.
- After you complete these experiments, you can attach the motor pulley to the motor. Instructions for doing this will be posted in the lab.

Sensor: Now we will assemble the sensor on the haptic device and calibrate it.

- In the lab, there are instructions for mounting your magnet and hall effect sensor on the haptic paddle.
- After you have mounted the magnet and sensor, measure the output of the sensor with an oscilloscope through the range of motion of the device. You want to obtain a monotonic relationship between the sensor output and the angle of the sector pulley θ. This means that the voltage only changes positively or negatively with θ. You should also get a difference in voltage of at least 1 volt. If the output range is less than this or the output does not vary monotonically with angle, then you need to reposition your magnet and/or sensor. See your lab instructor for help.
- The variation in sensor output is not quite linear, so we have written some software and a Matlab script that will compute a best-fit cubic spline to your data. You will need to record your values of A, B, C, and D (coefficients of the spline) for running your devices later. Before starting, make sure you have a blank diskette that you can bring to the lab to save your data. Hook up the sensor to the computer (see instructions in the lab - we can help you) and start the calibration program `C:\me161\calib.exe`. Using the special "Haptic device calibrator", a protractor with the center cut out, follow the directions given by the program. The program will ask you to gather data for

8 locations over the range of the handle motion. (Your handle angles will be between 230 degrees and 305 degrees on the protractor, with 270 degrees corresponding to vertical.) When you have the handle at the desired angle, have another group member hit `Enter` on the computer, then type in the corresponding angle in degrees. At the end, you will be asked to enter a filename for your data. The program will automatically add a `.m` extension to your filename.

Do this calibration, going from left to right with handle, three times. Use a different filename each time. The end result will be three `.m` files with your data. It is important that you use the same 8 angles for each run so keep track of your angles on a sheet of paper. Copy the three `.m` files onto your diskette on the `A:` drive and also take a copy of the Matlab data processing script `calib.m`. When you get to a computer with `Matlab`, put the `calib.m` and three `.m` files in the same folder and edit the `calib.m` file to load each of your data files. (It will be obvious how to edit it.) Then run the calibration script by typing `calib` at the Matlab prompt. The calibration script does the following:

- Averages your three runs (this is why you need to use the same angles each time)
- Computes the coefficients A, B, C, D of a best-fit 3rd order polynomial (a spline).
- Plots your three data runs and the spline fit.

If the results look good (i.e. fairly repeatable between the three runs and the spline seems to fit the data fairly well) you are DONE. Record your values of A, B, C, D for later use. If the results are not so great, you need to re-calibrate or adjust the position of your sensor and/or magnet. See an instructor for help.

Questions:

1. Motor: Provide your plot of torque vs. current and your calculation of K_m.
2. Give your best estimates for static and kinetic friction.
3. Provide your plot of voltage vs. speed and your calculation of K_v.
4. Sensor: Provide the plot of the output from the calibration script and the values of the coefficients of the spline fit A, B, C, and D.